Lecture 12:

Heat Transfer in the Ball

John Polking

PCMI, Summer 2003

This worksheet contains a set of graphs and pictures that illustrate how heat propagates inside a ball when the initial data
and boundary values are constant.

Fourier expansion of the solution

In what follows **u** is the solution, depending on the variables **r** (radius) and **t** (time), and parameters **k** (thermal
diffusivity), **u0** (initial temperature inside the ball), **f** (boundary temperature), and **a** (the ball's radius). The value
N is the upper limit of the sum in the Fourier series.

```
> u:=(r,t,k,u0,f,
  a,N)->f+2*(u0-f)*(a/(Pi*r))*sum((-1)^(n+1)*exp(-k*n^2*Pi^2*t/a^2
  )*sin(n*Pi*r/a)/n, n=1..N);
```

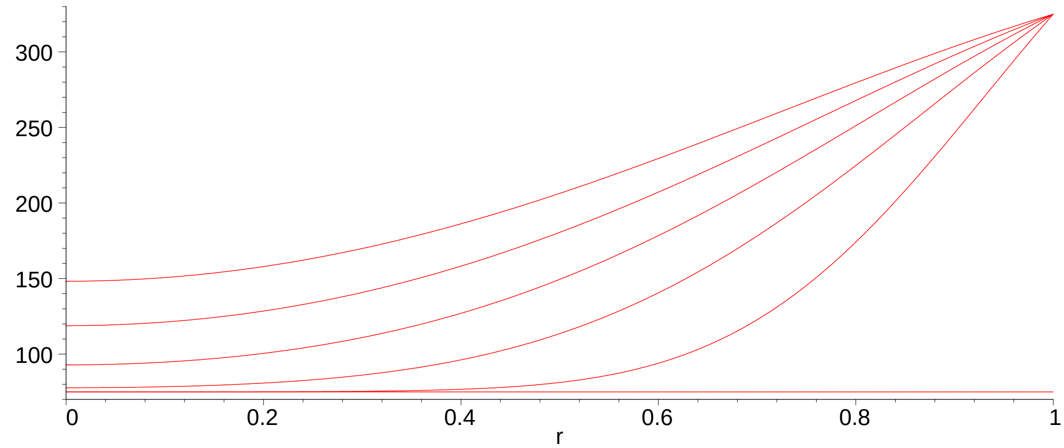
$$u := (r, t, k, u0, f, a, N) \rightarrow f + \frac{2\,(u0 - f)\,a \left(\sum_{n=1}^{N} \frac{(-1)^{(n+1)}\, \mathrm{e}^{\left(-\frac{k\,n^2\,\pi^2\,t}{a^2}\right)} \sin\left(\frac{n\,\pi\,r}{a}\right)}{n} \right)}{\pi\, r}$$

```
>
```

Temperature evolution inside the ball

We set **k=0.02**, **f=325**, **u0=75**, **a=1**, and **N=200**. The first graph displays the temperature profile for times **t= 0, 1, 2, 3, 4, 5.**

```
> with(plots):
> k:=0.02: u0:=75: f:=325: a:=1: N:=200:
> p[0]:=plot(75, r=0..1, numpoints=400):
> for j from 1 to 5 do
>   p[j]:=plot(u(r,j,k,u0,f,a,N), r=0..1, numpoints=400);
> end do:
> display([seq(p[j], j=0..5)]);
```

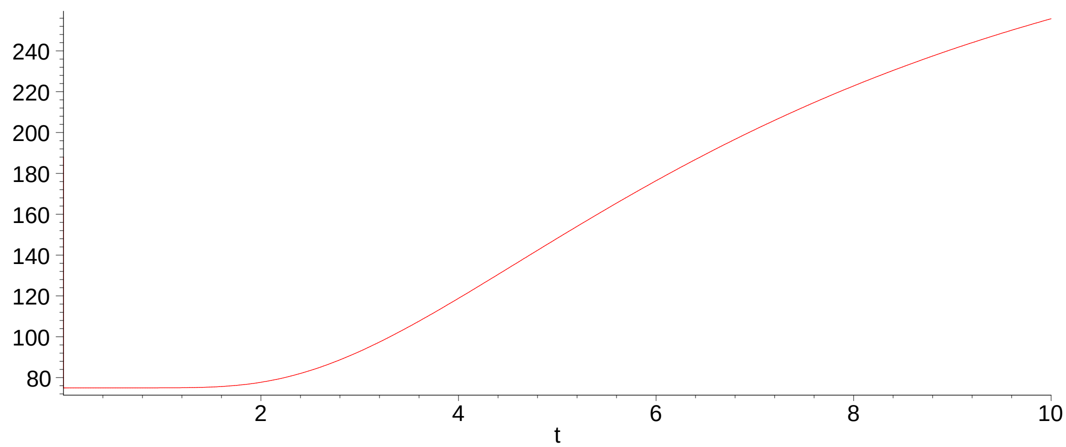


The second graph displays the temperature at the center of the ball against time. We need to input a formula for **w=u(0,t)** first.

```
> w:=t->f+2*(u0-f)*sum((-1)^(n+1)*exp(-k*n^2*Pi^2*t/a), n=1..N);
```

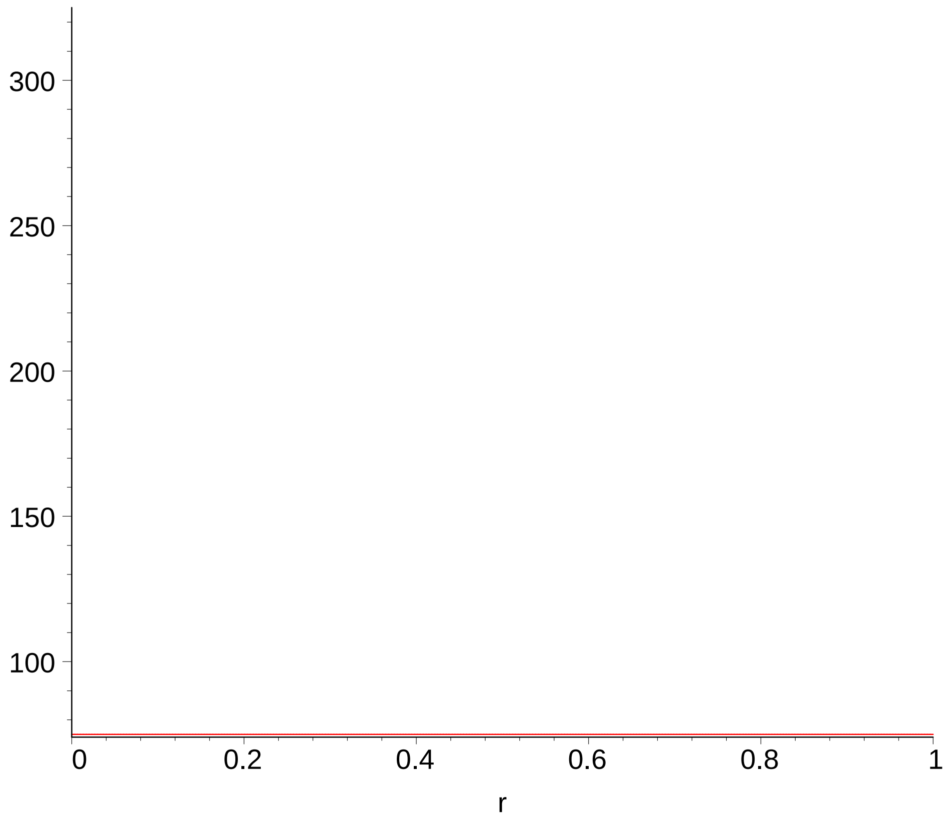
$$w := t \to f + 2\,(u0 - f)\left(\sum_{n=1}^{N} (-1)^{(n+1)}\, \mathbf{e}^{\left(-\frac{k\,n^2\,\pi^2\,t}{a}\right)}\right)$$

```
> plot(w(t),t=(.0001)..10, numpoints=400);
```



The last plot displays an animation of the temperature profile as times increases from zero to ten hours.

```
> q[0]:=plot(75, r=0..1, numpoints=300):
> for j from 1 to 40 do
>   q[j]:=plot(g(r,j/4), r=0..1, numpoints=300):
> end do:
> display([seq(q[j], j=0..40)], insequence=true);
```




```
>
```