Worksheet: **Lecture3.mws**

Lecture 3:

Laplace's Equation & Harmonic Functions

Jon Jacobsen

PCMI, Summer 2003

This worksheet contains some examples from the third lecture.

```
> restart:with(plots):
Warning, the name changecoords has been redefined
```

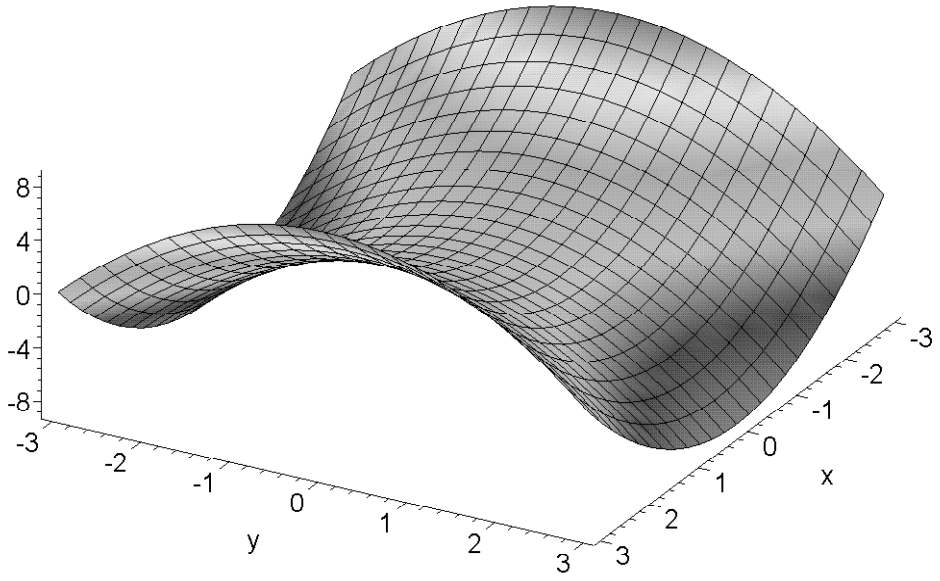
Complex Analysis

The real and imaginary parts of an analytic function are harmonic functions.

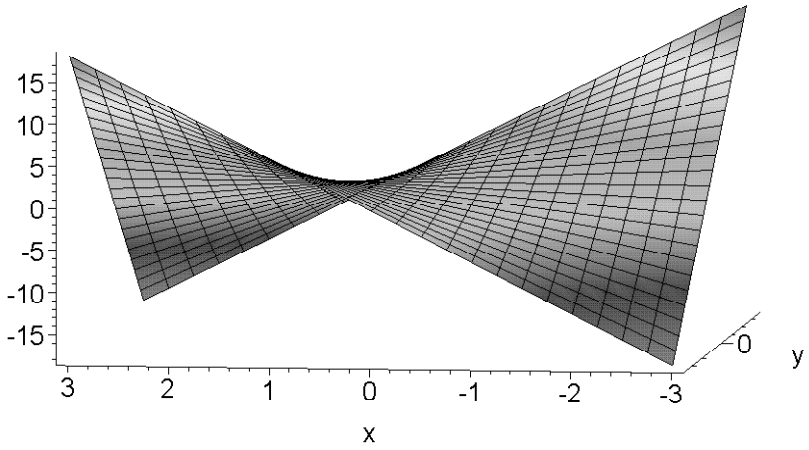
```
> f:=(x,y)->(x+I*y)^2;
```

$$f := (x, y) \to (x + y\,I)^2$$

```
> plot3d(Re(f(x,y)),x=-3..3,y=-3..3,axes=framed,shading=zhue);
```



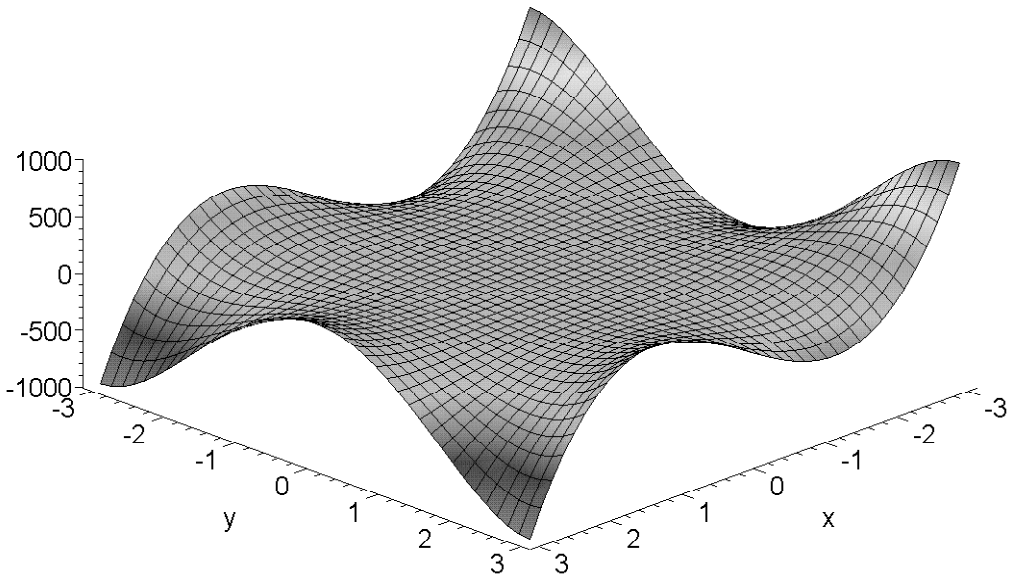
```
> plot3d(Im(f(x,y)),x=-3..3,y=-3..3,axes=framed,shading=zhue);
```



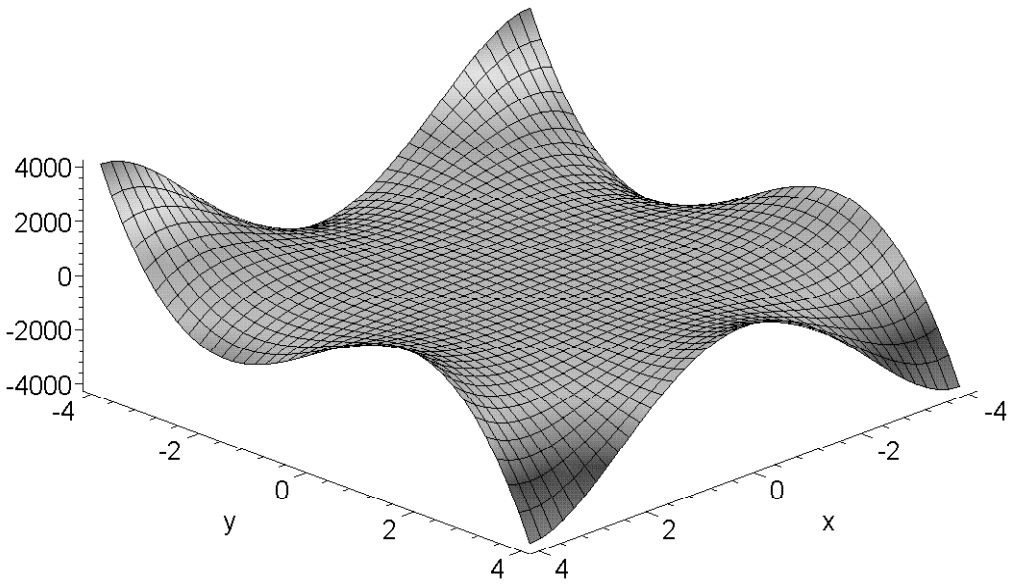

```
> f:=(x,y)->(x+I*y)^5;
```

$$f := (x, y) \to (x + y\,I)^5$$

```
> plot3d(Re(f(x,y)),x=-3..3,y=-3..3,axes=framed,shading=zhue,nump
  oints=1500);
```



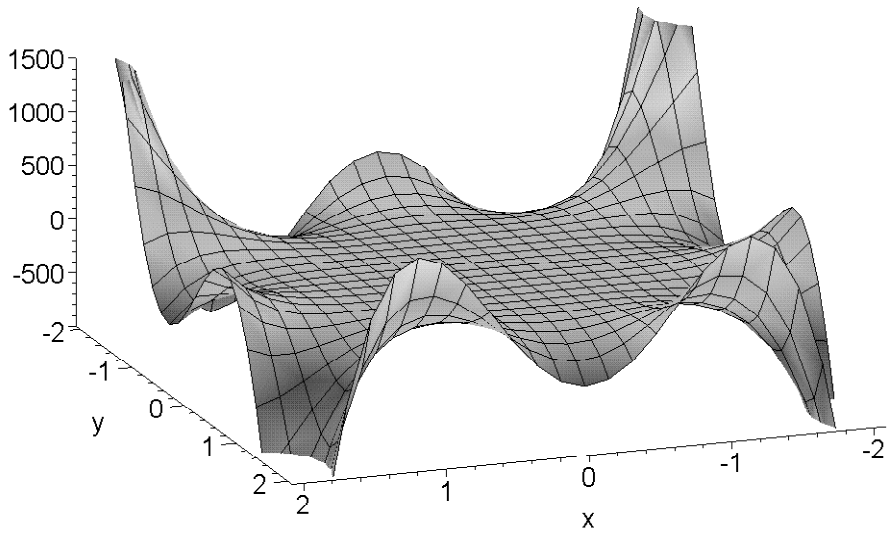
```
> plot3d(Im(f(x,y)),x=-4..4,y=-4..4,axes=framed,shading=zhue,nump
  oints=1500);
```




```
> f:=(x,y)->3*(x+I*y)^7-2*(x+I*y)^4;
```

$$f := (x, y) \to 3\,(x + y\,I)^7 - 2\,(x + y\,I)^4$$

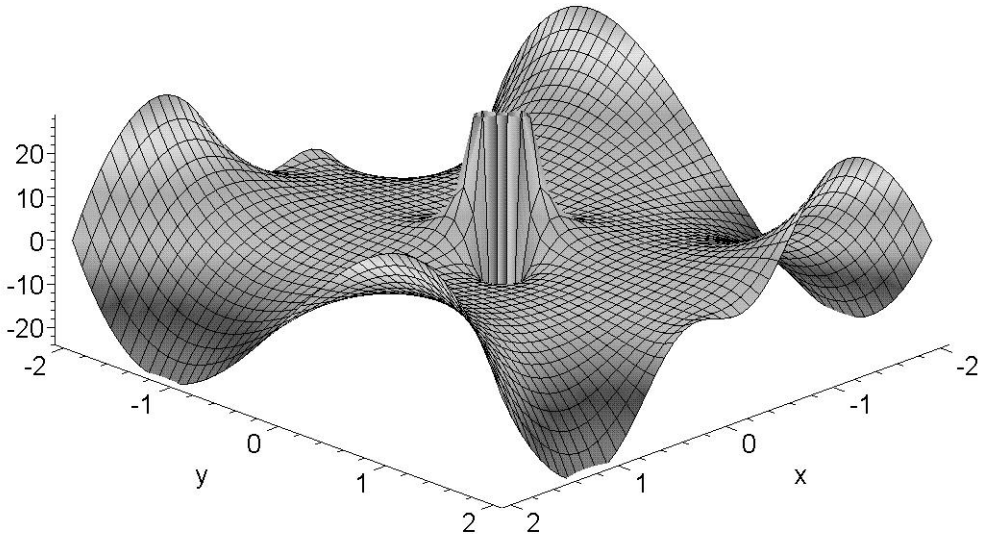
```
> plot3d(Im(f(x,y)),x=-2..2,y=-2..2,axes=framed,shading=zhue,view
  =-1000..1500);
```



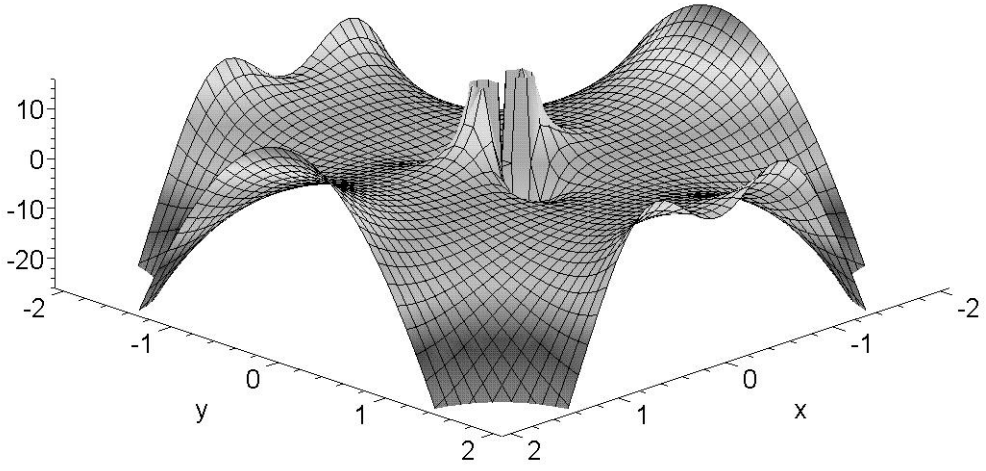
```
>
> f:=(x,y)->exp(-(x+I*y)^2)/((x+I*y)^2)+(x+I*y)^4;
```

$$f := (x, y) \to \frac{\mathbf{e}^{(-(x+y\,I)^2)}}{(x+y\,I)^2} + (x+y\,I)^4$$

```
> plot3d(Im(f(x,y)),x=-2..2,y=-2..2,axes=framed,shading=zhue,view
  =-24..29,numpoints=2000);
```




```
> plot3d(Re(f(x,y)),x=-2..2,y=-2..2,axes=framed,shading=zhue,view
  =-26..16,numpoints=2000);
```



Visualizing Perturbations

Here we use Maple to plot a surface and some perturbations

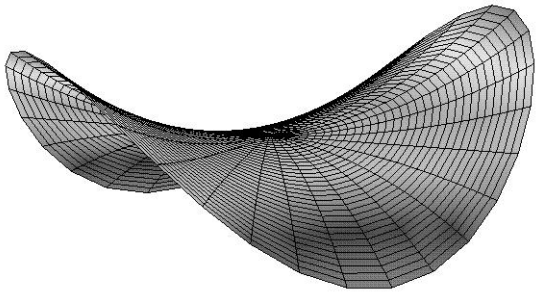
```
> addcoords(z_cylindrical,[z,r,theta],[r*cos(theta),r*sin(theta),z])
  ;
```

```
> g:=(r,t)->5+3/4*r^2*sin(2*t);
```

$$g := (r, t) \to 5 + \frac{3}{4} r^2 \sin(2\,t)$$

Here is a surface:

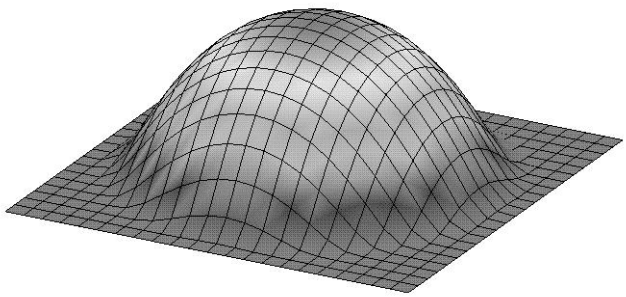
```
> plot3d(g(r,t),r=0..2,t=0..2*Pi,shading=zhue,grid=[35,35],coords=z_
  cylindrical,style=patchnogrid);
```



Here is an example of a "bump function" phi:

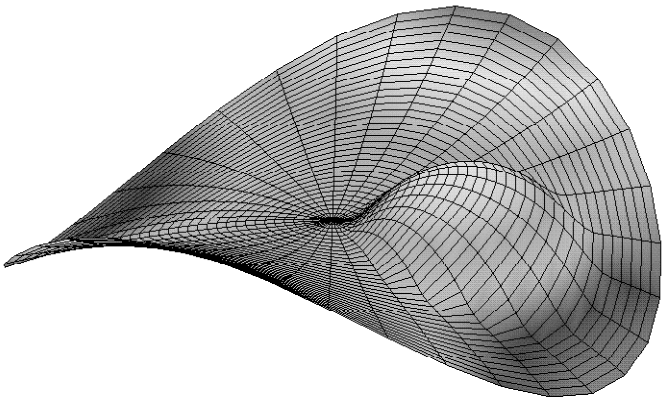
```
> p:=(x,y)->piecewise(x^2+y^2 >=1,0,exp(1/(x^2+y^2-1))):
```

```
> plot3d(.1*p(x,y),x=-1..1,y=-1..1,numpoints=500,style=patchnogrid,s
  hading=zhue);
```



Let's plot u plus the perturbation:

```
> animate3d([r*cos(t),r*sin(t),g(r,t)+7.5*cos(eps)*p(r*cos(t)-.9,r*sin(t)+.5)],r=0..2.3,t=0..2*Pi,eps=0..2*Pi,axes=none,shading=zhue,grid=[35,35],style=patch);
```



```
> plot3d(g(r,t)-8.5*p(r*cos(t)-.8,r*sin(t)+.5),r=0..2.3,t=0..2*Pi,axes=none,shading=zhue,grid=[35,35],coords=z_cylindrical,style=patchnogrid);
```