PCMI-UFP

LAB 5: The Diffusion Equation: Solution by Separation of Variables

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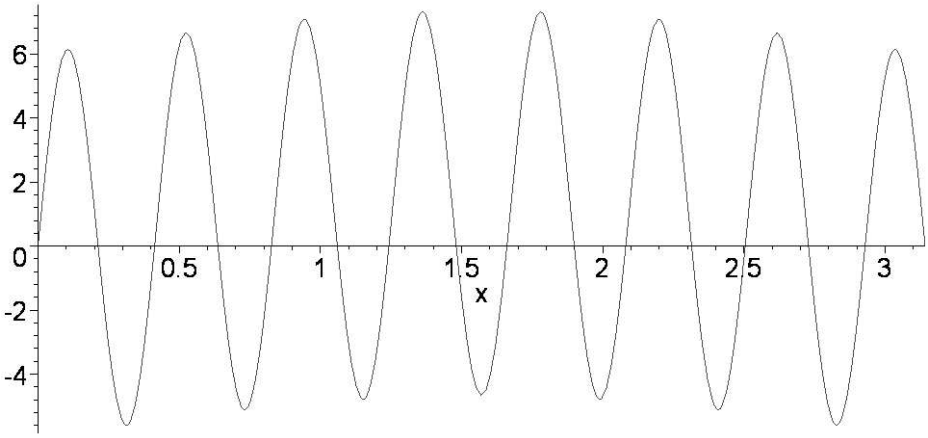
Solving the Diffusion Equation given f(x) = 6 sin(15x) + 4/3sin(x)

```
> restart;with(plots):
Warning, the name changecoords has been redefined
```

```
> f:=x -> 6*sin(15*x) + 4/3*sin(x);plot(f(x), x=0..Pi);
```

$$f := x \to 6\sin(15\,x) + \frac{4}{3}\sin(x)$$



Set k=1

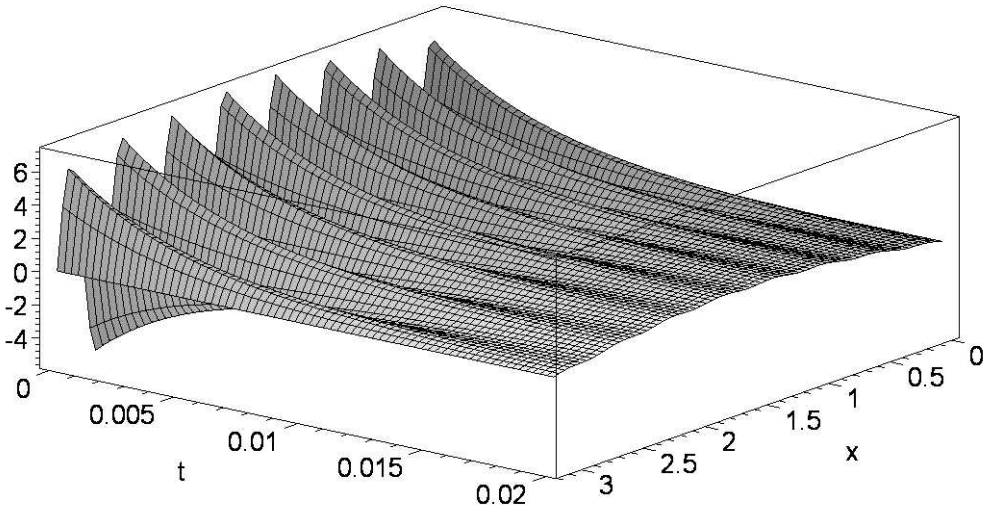
```
> u:=(x,t)-> 6*exp(-15^2*t)*sin(15*x) + 4/3*exp(-t)*sin(x);
```

$$u := (x, t) \to 6\,\mathbf{e}^{(-225\,t)}\sin(15\,x) + \frac{4}{3}\mathbf{e}^{(-t)}\sin(x)$$

```
> ut:= diff(u(x,t),t):uxx:= diff(u(x,t),x$2):simplify(ut-uxx);
```

0

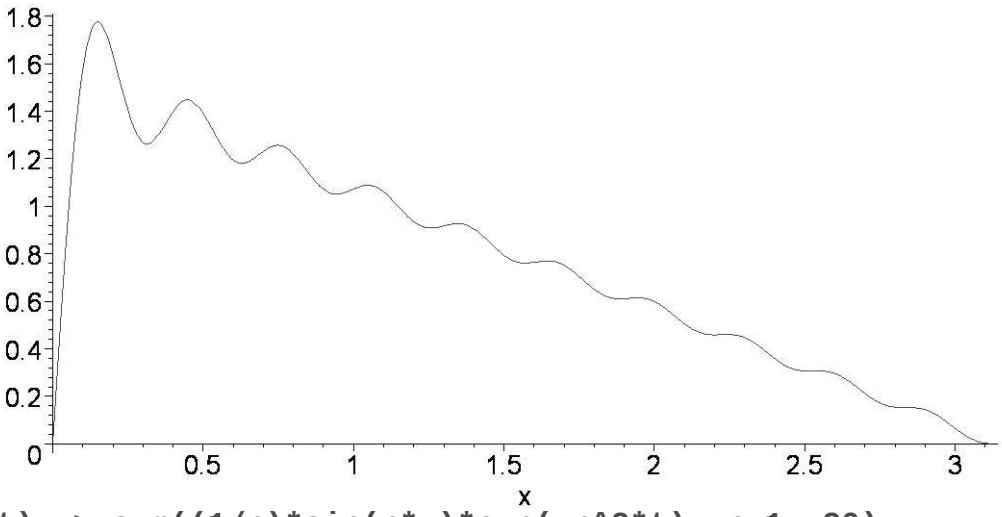
```
> plot3d(u(x,t), x=0..Pi,t=0..0.02,numpoints=5000, axes=boxed);
```



Solving the Diffusion Equation given a Fourier Series

```
> f:= x -> sum((1/n)*sin(n*x), n=1..20);plot(f(x), x=0..Pi);
```

$$f := x \to \sum_{n=1}^{20} \frac{\sin(n\,x)}{n}$$




```
> u:=(x,t) -> sum((1/n)*sin(n*x)*exp(-n^2*t), n=1..20);
```

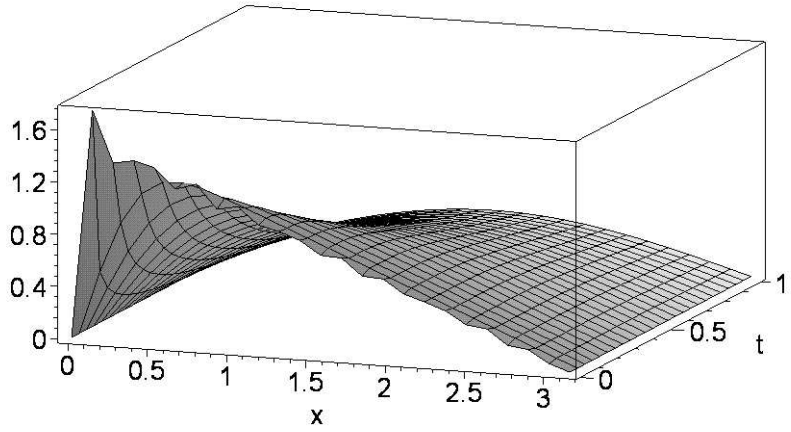
Here we set kappa = 1 in the differential equation

$$u := (x, t) \to \sum_{n=1}^{20} \frac{\sin(n\,x)\,\mathbf{e}^{(-n^2\,t)}}{n}$$

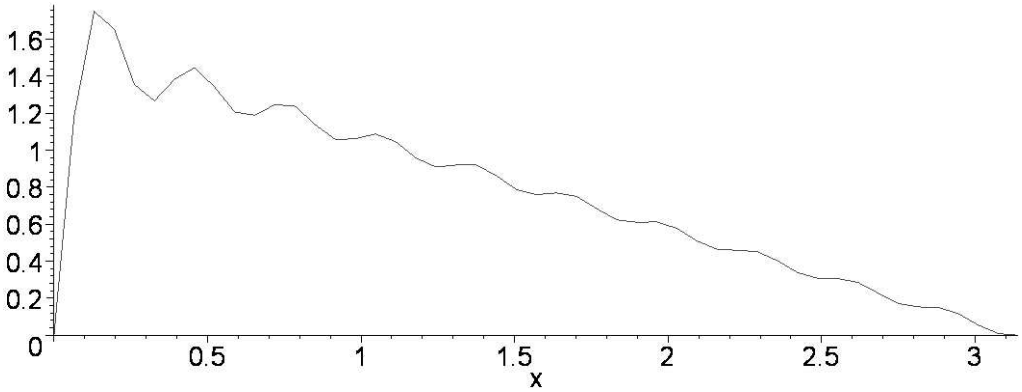
```
> ut:= diff(u(x,t),t):uxx:= diff(u(x,t),x$2):simplify(ut-uxx);
>
```

0

```
> plot3d(u(x,t),x=0..Pi, t=0..1, axes=boxed);
```




```
> animate(u(x,t),x=0..Pi, t=0..3);
```

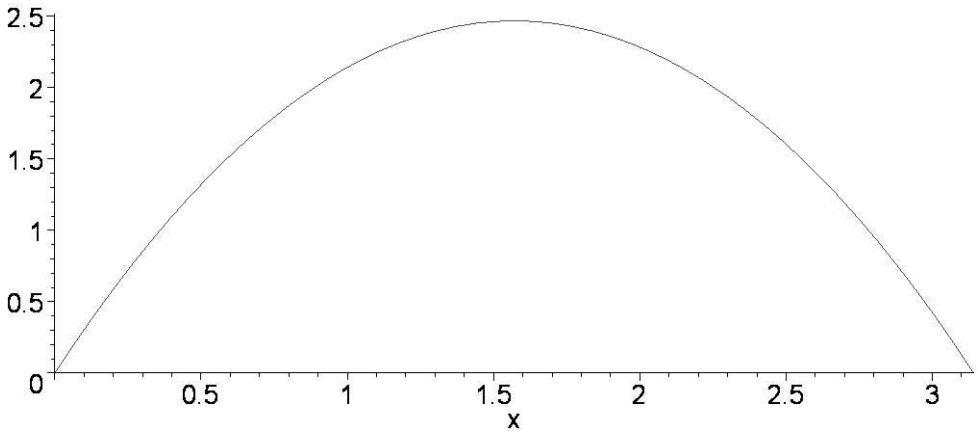


Finding Fourier Coefficients

```
> assume(n, integer): assume(N,integer):
```

Set f and calculate its Fourier coefficients for sin

```
> f:= x->(Pi-x)*x;plot(f(x),x=0..Pi);
  a := [seq(int(f(x)*sin(n*x),x=0..Pi)*2/(Pi),n=1..17)];
```

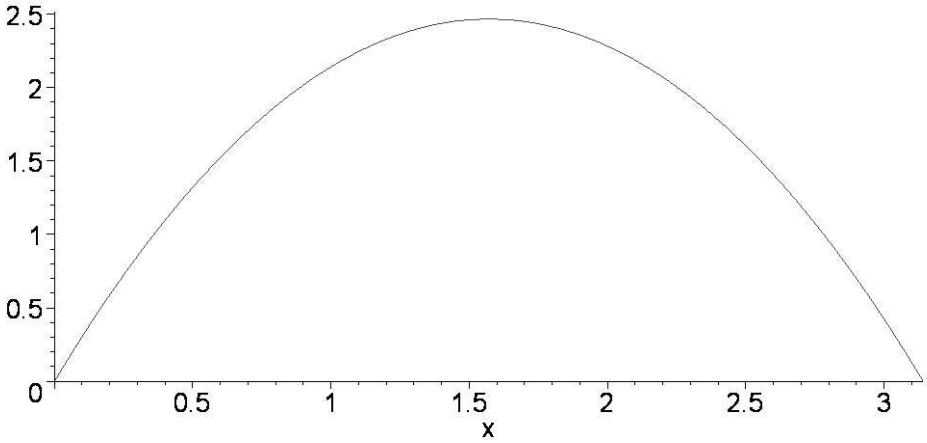


$$a := \left[\frac{8}{\pi}, 0, \frac{8}{27\,\pi}, 0, \frac{8}{125\,\pi}, 0, \frac{8}{343\,\pi}, 0, \frac{8}{729\,\pi}, 0, \frac{8}{1331\,\pi}, 0, \frac{8}{2197\,\pi}, 0, \frac{8}{3375\,\pi}, 0, \frac{8}{4913\,\pi}\right]$$

Form the sin Fourier Series of f

```
> A:= x-> sum(a[k]*sin(k*x),k=1..20);
  plot(A(x), x=0..Pi);
```

$$A := x \to \sum_{k=1}^{20} a_k \sin(k\,x)$$

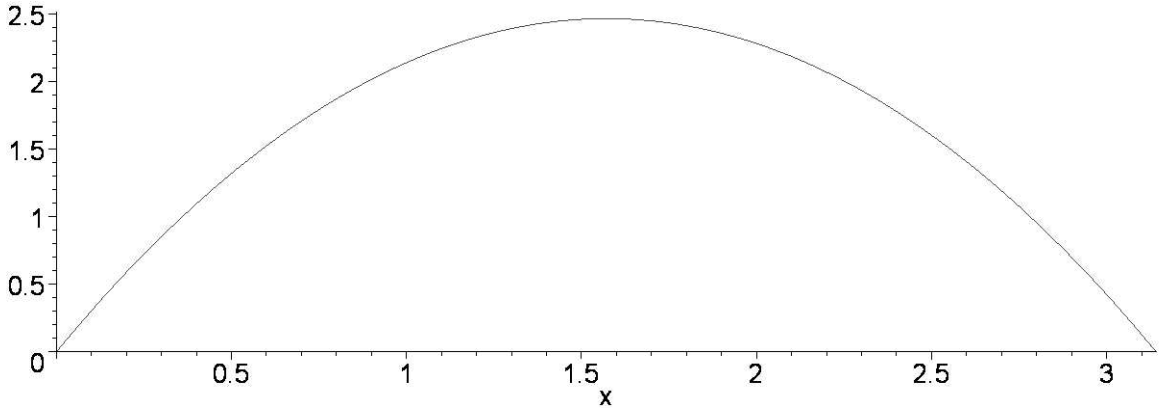


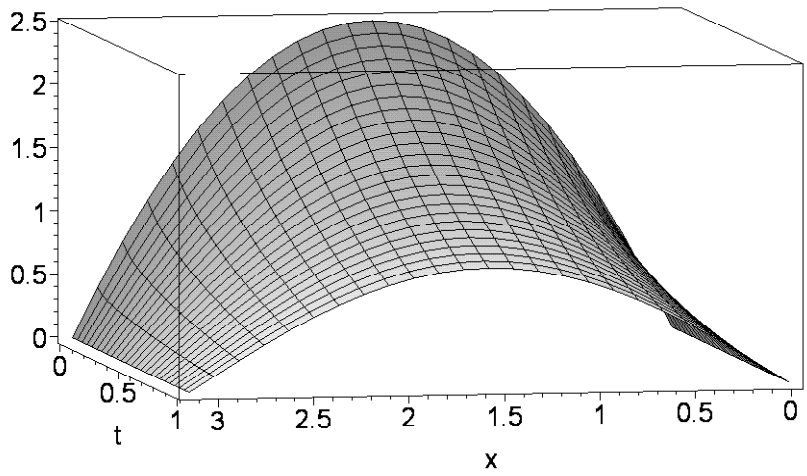
Solving the differential equation with the calculated Fourier Coefficients

Form the solution to the Diffusion Equation using the initial value f
Set k=1

```
> u:= (x,t)-> sum(exp(-k^2*t)*a[k]*sin(k*x),k=1..20);
  animate(u(x,t),x=0..Pi,t=0..1);
  plot3d(u(x,t),x=0..Pi,t=0..1, axes=boxed);
```

$$u := (x, t) \to \sum_{k=1}^{20} \mathbf{e}^{(-k^2\,t)}\, a_k \sin(k\,x)$$





One more example

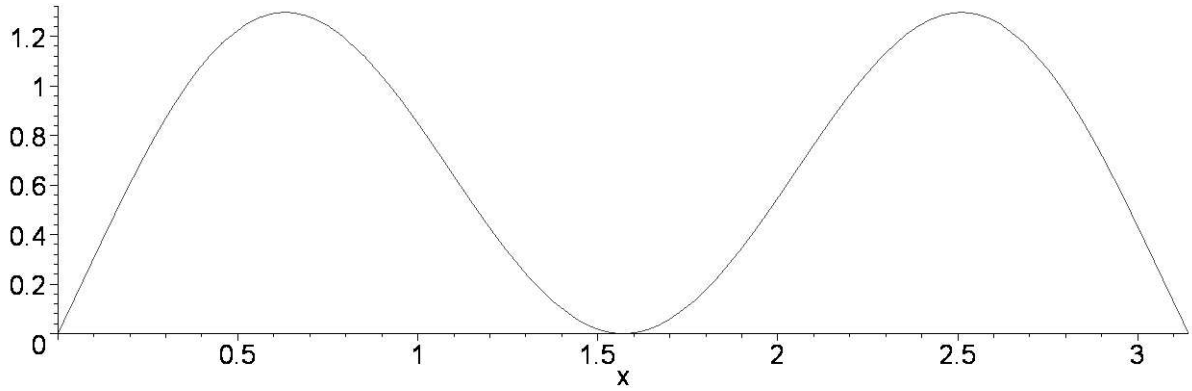
```
> assume(n, integer): assume(N,integer):
```

```
> f:= x->(sin(x))^2+ sin(3*x);
  plot(f(x),x=0..Pi);

  b := [seq(int(f(x)*sin(n*x),x=0..Pi)*2/(Pi),n=1..20)];

>
```

$$f := x \to \sin(x)^2 + \sin(3\,x)$$

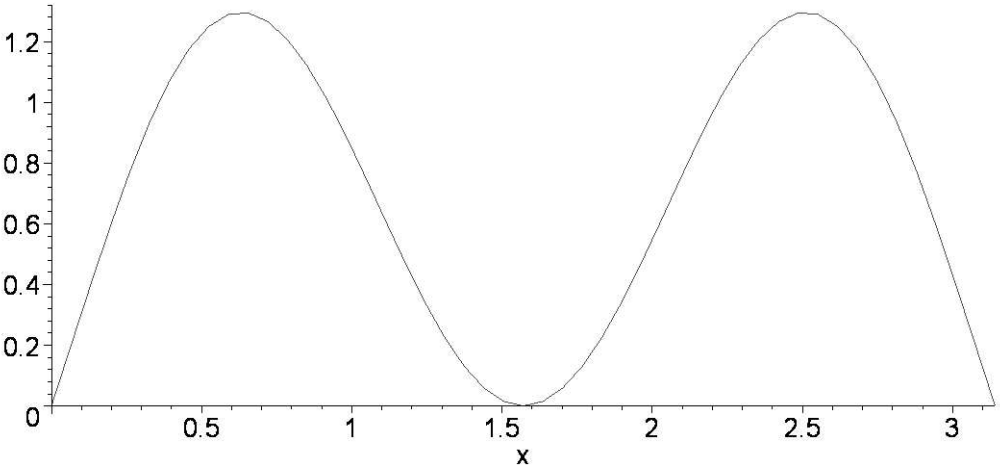


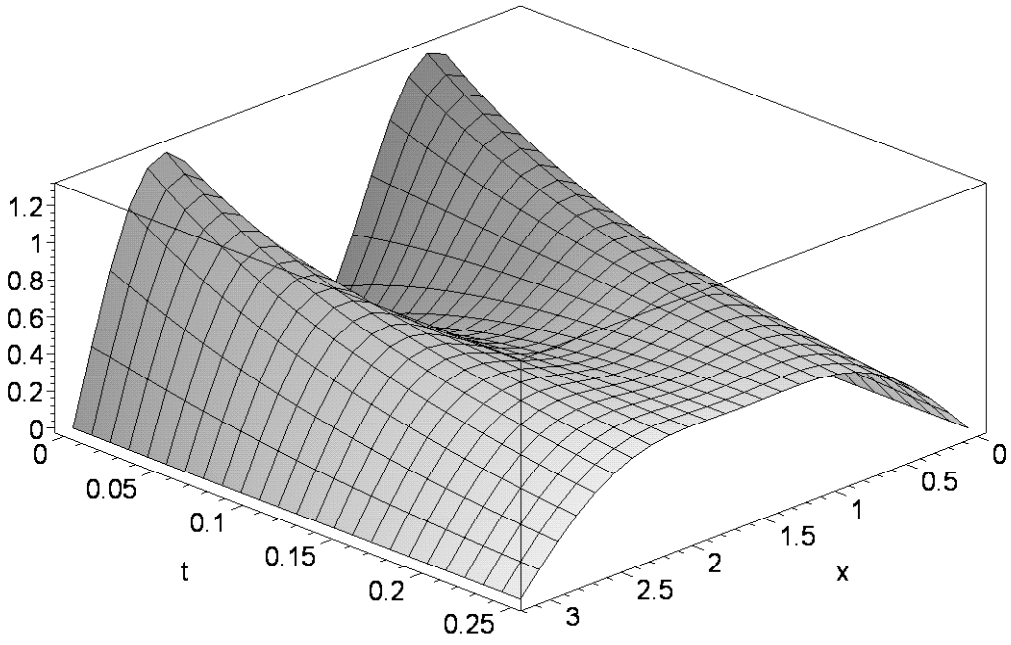
$$b := \left[\frac{8}{3\,\pi}, 0, \frac{2\left(-\frac{4}{15}+\frac{\pi}{2}\right)}{\pi}, 0, -\frac{8}{105\,\pi}, 0, -\frac{8}{315\,\pi}, 0, -\frac{8}{693\,\pi}, 0, -\frac{8}{1287\,\pi}, 0, -\frac{8}{2145\,\pi}, 0, -\frac{8}{3315\,\pi}, 0, -\frac{8}{4845\,\pi}, 0, -\frac{8}{6783\,\pi}, 0\right]$$

Form the solution to the Diffusion Equation using the initial value g
Set k=1

```
> u:= (x,t)-> sum(exp(-k^2*t)*b[k]*sin(k*x),k=1..20);
  animate(u(x,t),x=0..Pi,t=0..0.5);
  plot3d(u(x,t),x=0..Pi,t=0..0.25, axes=boxed);
```

$$u := (x, t) \to \sum_{k=1}^{20} \mathbf{e}^{(-k^2 t)} b_k \sin(k x)$$



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