Worksheet: **Lecture8.mws**

Lecture 8: Sturm-Liouville, Part I

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This worksheet contains the example from the eighth lecture.

Heat Equation with Radiation Boundary Conditions

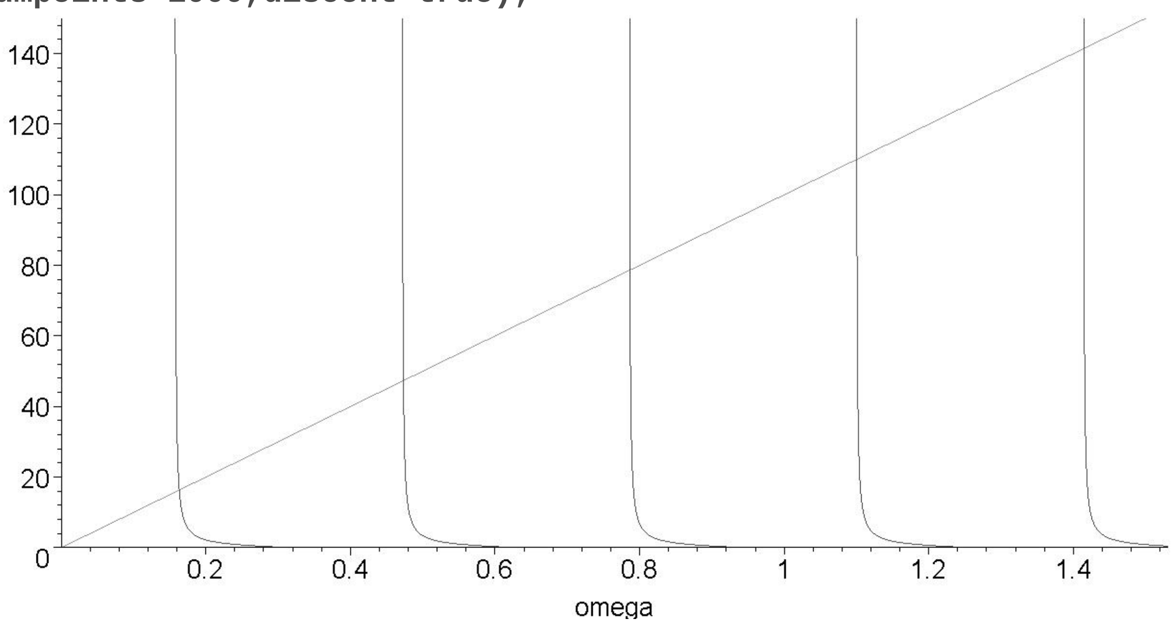
```
> restart:
> with(plots):
Warning, the name changecoords has been redefined
> K:=1: h:=1/100: T1:=75: T2:=-125: L:=10: N:=20:
> u_s:=x->(h*(T2-T1)/(1+h*L))*x+T1;
```

$$u\_s := x \rightarrow \frac{h\,(T2 - T1)\,x}{1 + h\,L} + T1$$

```
> surprise:=plot(u_s(x),x=0..L):
> F:=x->T1-u_s(x);
```

$$F := x \rightarrow T1 - \mathrm{u\_s}(x)$$

```
> plot(F(x),x=0..L):
> plot([-tan(omega*L),omega/h],omega=0..(2*N-1)*Pi/(8*L),0..150,n
  umpoints=1000,discont=true);
```



```
> ints:=seq((2*n-1)*Pi/(2*L)..(2*n+1)*Pi/(2*L),n=1..N);
```

$$ints := \frac{\pi}{20}..\frac{3\pi}{20}, \frac{3\pi}{20}..\frac{\pi}{4}, \frac{\pi}{4}..\frac{7\pi}{20}, \frac{7\pi}{20}..\frac{9\pi}{20}, \frac{9\pi}{20}..\frac{11\pi}{20}, \frac{11\pi}{20}..\frac{13\pi}{20}, \frac{13\pi}{20}..\frac{3\pi}{4},$$
$$\frac{3\pi}{4}..\frac{17\pi}{20}, \frac{17\pi}{20}..\frac{19\pi}{20}, \frac{19\pi}{20}..\frac{21\pi}{20}, \frac{21\pi}{20}..\frac{23\pi}{20}, \frac{23\pi}{20}..\frac{5\pi}{4}, \frac{5\pi}{4}..\frac{27\pi}{20}, \frac{27\pi}{20}..\frac{29\pi}{20},$$
$$\frac{29\pi}{20}..\frac{31\pi}{20}, \frac{31\pi}{20}..\frac{33\pi}{20}, \frac{33\pi}{20}..\frac{7\pi}{4}, \frac{7\pi}{4}..\frac{37\pi}{20}, \frac{37\pi}{20}..\frac{39\pi}{20}, \frac{39\pi}{20}..\frac{41\pi}{20}$$

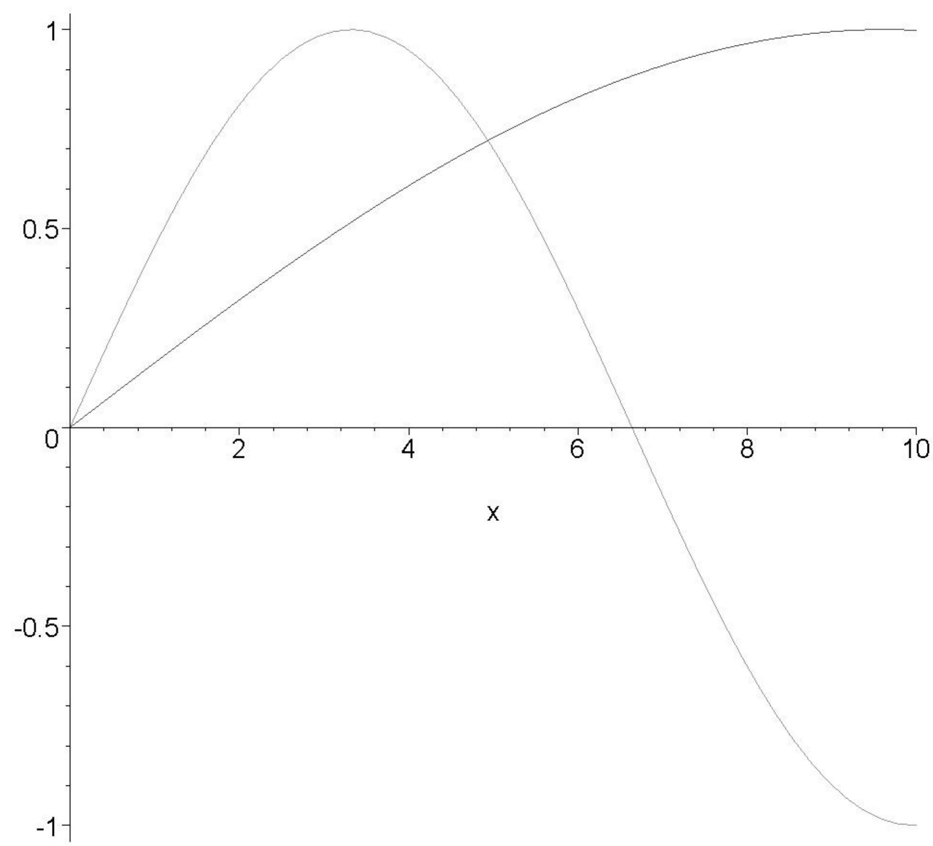
```
> Omega:=seq(fsolve(-tan(omega*L)=omega/h,omega=ints[n]),n=1..N);
```

Ω := 0.1631994527, 0.4733511802, 0.7866692772, 1.100466110, 1.414423684, 1.728454505, 2.042524811, 2.356618824, 2.670728184, 2.984848045, 3.298975410, 3.613108321, 3.927245448, 4.241385854, 4.555528861, 4.869673965, 5.183820786, 5.497969029, 5.812118463, 6.126268906

```
> X:=n->sin(Omega[n]*x);
```

$$X := n \rightarrow \sin(\Omega_n\, x)$$

```
> plot([X(1),X(2)],x=0..L);
```




```
> int(X(1)*X(2),x=0..L);
```

$$0.67331\ 10^{-9}$$

```
> C:=n->int(F(x)*sin(Omega[n]*x),x=0..L)/int(sin(Omega[n]*x)^2,x=0..L);
```

$$C := n \to \frac{\int_0^L \mathrm{F}(x)\,\sin(\Omega_n\,x)\,dx}{\int_0^L \sin(\Omega_n\,x)^2\,dx}$$

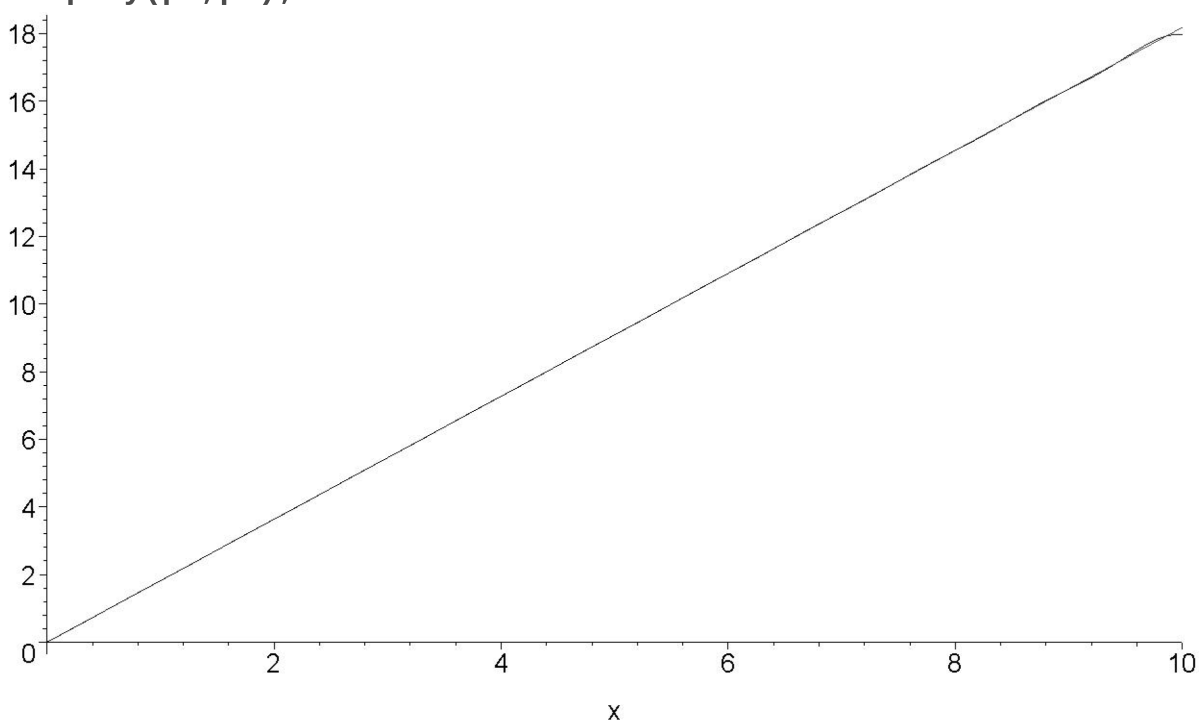
```
> c:=seq(C(n),n=1..N);
```

c := 14.44974861, -1.776900294, 0.6452669612, -0.3300124268, 0.1998357000, -0.1338417200, 0.09585527458, -0.07201105398, 0.05607079795, -0.04489152384, 0.03675022210, -0.03063819026, 0.02593309564, -0.02223407633, 0.01927349359, -0.01686711973, 0.01488480147, -0.01323245249, 0.01184070371, -0.01065751046

```
> v:=(x,t)->sum(c[n]*exp(-Omega[n]^2*K*t)*sin(Omega[n]*x),n=1..20);
```

$$v := (x, t) \to \sum_{n=1}^{20} c_n\,\mathbf{e}^{\left(-\Omega_n^{\,2}\,K\,t\right)}\,\sin(\Omega_n\,x)$$

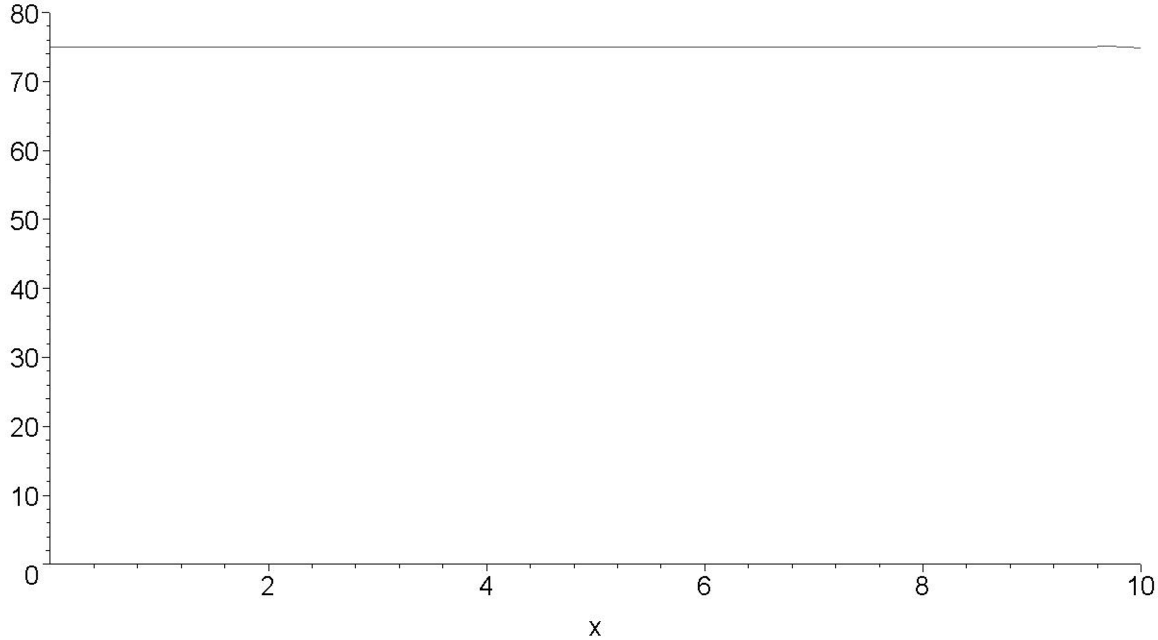
```
> p1:=plot(v(x,0),x=0..L,color=blue):
> p2:=plot(F(x),x=0..L,color=red):
> display(p1,p2);
```



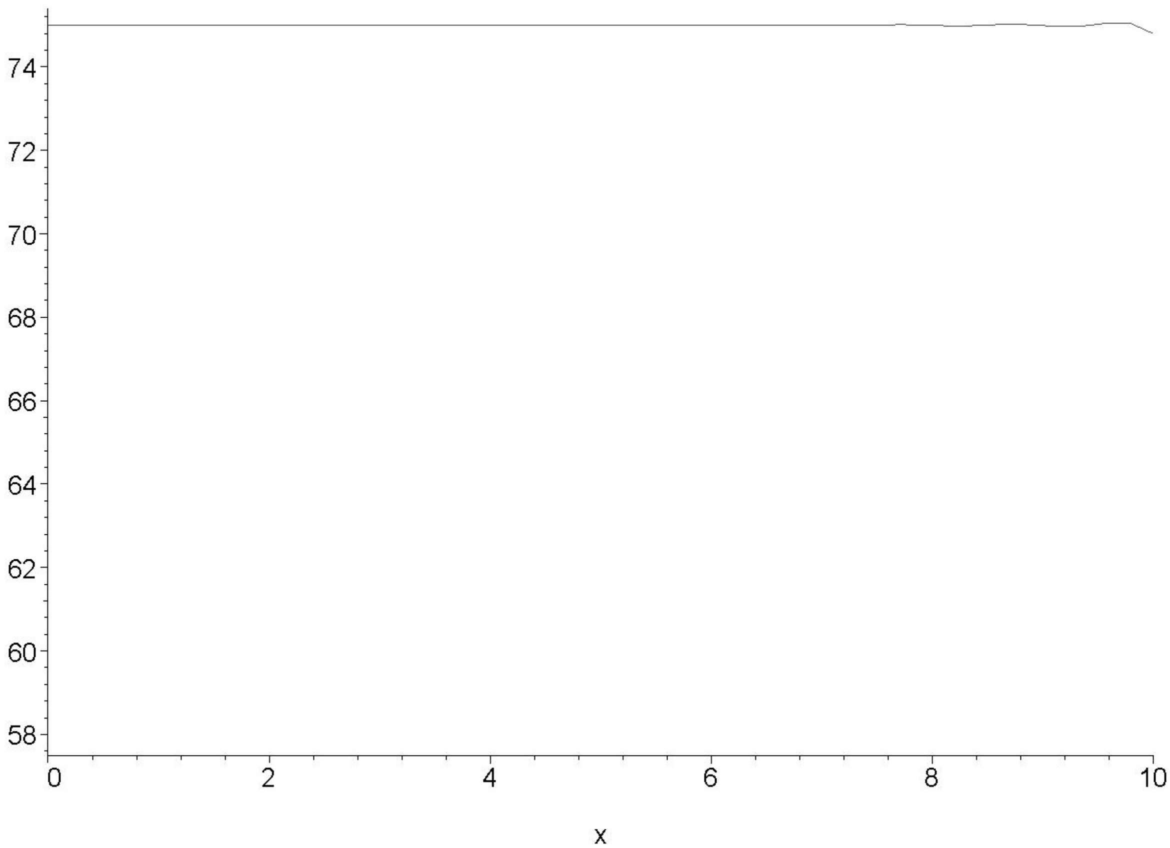
```
> u:=(x,t)->v(x,t)+u_s(x);
```

$$u := (x, t) \to \mathrm{v}(x, t) + \mathrm{u\_s}(x)$$

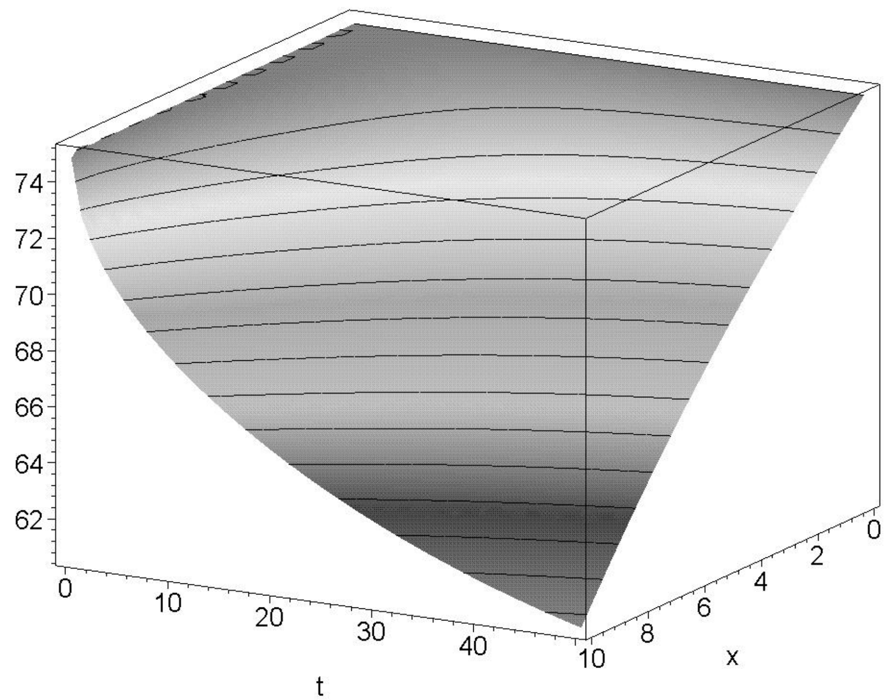
```
> plot(u(x,0),x=0..L,view=[0..L,0..80]);
```




```
> animate(u(x,t),x=0..L,t=0..100,frames=100);
```



```
> plot3d(u(x,t),x=0..L,t=0..50,axes=boxed,grid=[50,50],style=patc
  hcontour,shading=zhue);
```




```
> uend:=plot(u(x,200),x=0..L,color=blue):
> display(surprise,uend);
```