PCMI | **Pizza & Problem Solving # 2** | Andrew J. Bernoff
Tuesday, July 17, 2007 | | Harvey Mudd College

Mathematical Games

This problem session is modelled after the HMC Putnam Preparation Problem Solving Seminar co-led with Francis Su. Additional resources (and this problem set) can be found at:

`http://www.math.hmc.edu/~ajb/PCMI/problem_solve.html`

B1: In the game of NIM a pile of N chips is diminished by two players who alternately remove a number of chips, where the number is chosen from a set M. The person who takes the last chip wins. Determine the values of N for which the first player wins if:

(a) Powers of Two: $M = \{1, 2, 4, 8 \cdots\}$ (b) (Hard!) $M = \{1, 3, 8\}$ (Engel)

B2: (a) Two players play a game in which the first player places a king on an empty 8×8 chessboard, and then, starting with the second player, they alternate moving the king (in accord with the rules of chess) to a square that has not been previously occupied. The player who moves last wins. Which player has a winning strategy?

(b) Suppose a knight is used instead of a king, which player has a winning strategy now?

(c) Suppose a 5×5 chessboard is used instead; answer parts (a) and (b) above. (Engel)

B3: Two players alternately draw diagonals between vertices of a regular polygon. They may connect two vertices if they are non-adjacent (i.e. not a side) and if the diagonal formed does not cross any of the previous diagonals formed. The last player to draw a diagonal wins.

(a) Who wins if the polygon is a pentagon?

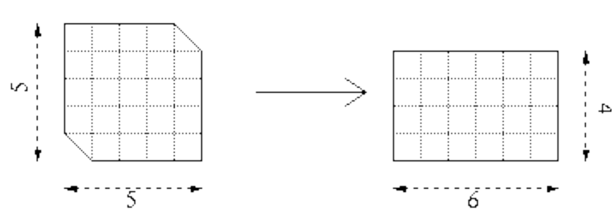
(b) Who wins if the polygon is a hexagon?

(c) Who wins if the polygon has a thousand vertices?

B4: Two people take turns breaking up a rectangular chocolate bar that is 4×6 squares in size. You can only break the bar along a division between the squares and only in a straight line. So, for example, the first person could break the bar into two 4×3 pieces and the second could break one of the 4×3 pieces into a 4×1 piece and a 4×2 piece. When the bar has been divided into single squares, the person who made the last division wins all the chocolate. Assuming you like chocolate, would you rather go first or second? (Zeitz)

And for a little bit of variety...

B5: Paper & Scissors You have a 5×5 piece of paper and a pair of scissors. Two diagonally opposite corners of this paper are truncated as shown in the diagram below. Show how to cut up the 5×5 paper into two pieces, that can be arranged to form a 6×4 rectangle. (Wu)



Hints:

1. In both cases, consider a small number of chips.
2. (a) Think about covering the board in 2×1 dominos. Why does this help?
3. (a) and (b) can be done by brute force. In (c) can you explot a symmetry?
4. Try a few smaller candy bars first; what is the same no matter how you play the game?