PCMI-UFP
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MAPLE Challenge Problems

These problems are designed to encourage the use of MAPLE, other computational aids, and web resources.

A1: Consider the function

$$f_N(x) = \sum_{n=1}^{N} \frac{\sin(nx)}{n} \qquad 0 < x < \pi \quad .$$

a) Suppose these are partial sums of a Fourier series; can you guess the function $f(x)$ they are converging to?

b) Determine $\lim_{N\to\infty} f_N(\pi/2)$. It may help if you evaluate $\sin(nx)$ and let MAPLE evaluate the resulting infinite series.

c) Determine the value x_* where $f_N(x)$ reaches a maximum. You should be able to find the answer explicitly.

d) Determine $\lim_{N\to\infty} f_N(x_*)$. You might try evaluating the limit as a Riemann sum.

e) Relate your result in (d) to Gibb's phenomena.

A2: The sequence a_n satisfies the recurrence relationship

$$a_1 = 1 \qquad a_2 = 1 \qquad a_3 = 4 \qquad a_{n+3} = 2a_{n+2} + 2a_{n+1} - a_n \quad \text{for} \quad n = 1, 2, 3, \ldots$$

Prove that a_n is a perfect square for all n.

A3: (a) What are the last two digits of 3^{2003} ? $3^{2003^{2003}}$?

(b) What is the last non-zero digit of 2003! ?

A4: Simplify the expression:

$$x = \left(2 + \frac{10}{3\sqrt{3}}\right)^{1/3} + \left(2 - \frac{10}{3\sqrt{3}}\right)^{1/3}$$

(Hess)

A5: Find positive real numbers $x_1, x_2, \ldots x_n$ such that their sum is 2003 and their product is as large as possible.

A6: Let $x_1 = 1$ and for $m \geq 1$ let $x_{m+1} = (m+3/2)^{-1} \sum_{k=1}^{m} x_k x_{m+1-k}$. Evaluate $\lim_{m\to\infty} x_m/x_{m+1}$.
(American Mathematical Monthly)

A7: What arrangement of the digits $0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 0$ into the product of five two digit numbers yields the largest product? For example, $10 \times 23 \times 45 \times 67 \times 89 = 61,717,050$.
(Kornhauser, Velleman & Wagon)