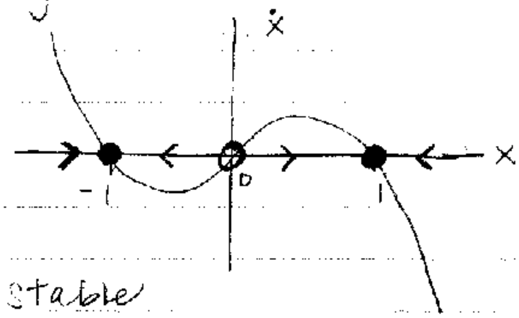
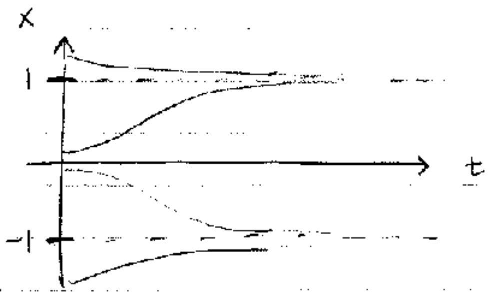
DAY 2

Systems, Flows and Stability ; Solutions

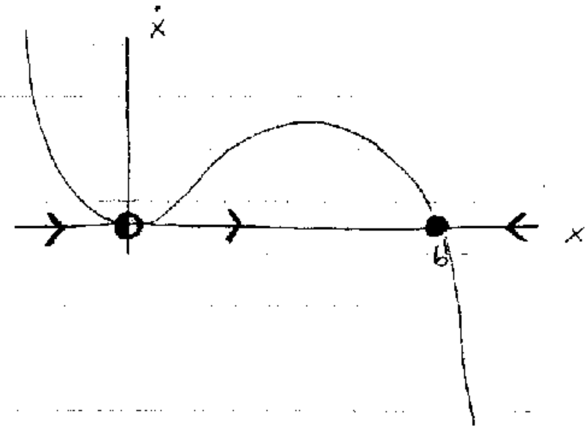
1. (a) $\dot{x} = x - x^3$
$= x(1-x^2)$



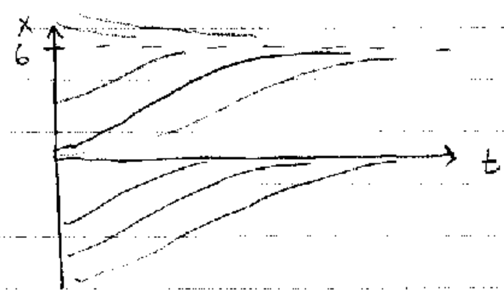
fixed points: $0, \pm 1$
0 : unstable ± 1 : stable



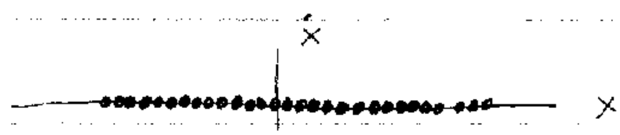
(b) $\dot{x} = x^2(b-x)$



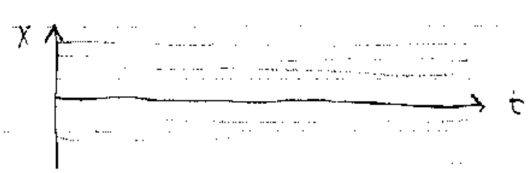
fixed points $0, b$
0 : half stable
b : stable



(c) $\dot{x} = 0$

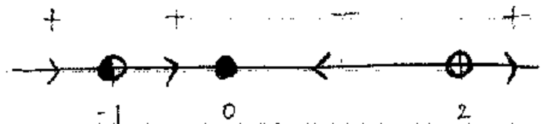


No flow - all points are fixed points



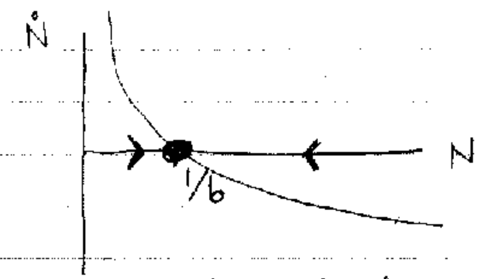
Systems, Flow & Stability   SOLUTIONS - cont'd

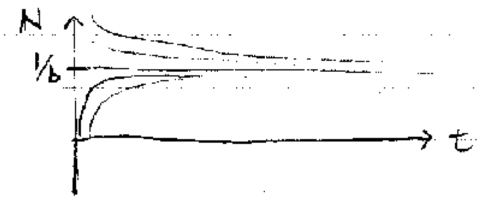
2.



$x(x+1)^2(x-2)$

3. $\dot{N} = -aN \ln(bN)$





b is the reciprocal of the "optimal" tumor size for the given conditions. a is an intrinsic growth rate.

4. (a) (i) a_1, a_2 are the growth rates of normal & tumor cells respectively.

(ii) b_1, b_2 are the normal & tumor cells respective carrying capacities.

(iii) c_1, c_2 are competition rates

These parameter values are all positive since the negative signs indicate decrease.

(b) The growth rates could be determined by examining a fixed number of cells w/ an infinite nutrient supply. (Normal and tumor cells each in separate dishes!) The carrying capacity could be determined by a similar experiment w/ limited nutrient amount. Competition rates could be determined by labelling normal cells (w/ a detectable tag) introducing different ratios of tumor to normal cells. Tumor cells would kill amounts of normal cells which would be measured by the tag. (A "tag" example is chromium which could be detected/measured by centrifugation.)