

# Non-dimensionalization Exercises

## For You!

*Exercise 1.*<sup>1</sup> Consider the logistic equation  $N = rN(1 - N/K)$ , with initial condition  $N(0) = N_0$ .

(a) The system has three dimensional parameters  $r$ ,  $K$ , and  $N_0$ . Find the dimensions of each of these parameters.

(b) Show that the system can be rewritten in the dimensionless form

$$\frac{dx}{d\tau} = x(1 - x); \quad x(0) = x_0$$

for appropriate choices of the dimensionless variables,  $x$ ,  $x_0$  and  $\tau$ .

(c) Find a different non-dimensionalization in terms of variables  $u$  and  $\tau$ , where  $u$  is chosen so that the initial condition is always  $u_0 = 1$ .

(d) Can you think of any advantage of one non-dimensionalization over the other?

*Exercise 2.*<sup>2</sup> The temperature  $T(t)$  of a chemical sample in a furnace at time  $t$  is governed by the initial value problem

$$\frac{dT}{dt} = qe^{-\theta/T} - k(T - T_f); \quad T(0) = T_0$$

where  $T_0$  is the initial temperature of the sample,  $T_f$  is the temperature in the furnace, and  $q$ ,  $k$ , and  $\theta$  are positive constants.

(a) What must be the dimensions of the constants  $q$ ,  $k$  and  $\theta$ ?

(b) Reduce the problem to dimensionless form using  $T_f$  as the temperature scale. Can you choose a time scale so that the heat loss term is large compared to the heat generated by the reaction?

### References

1. Logan, J.D. *Applied Mathematics*, 2nd Edition. John Wiley and Sons, Inc., 1997.
2. Strogatz, S. *Nonlinear Dynamics and Chaos*. Perseus Books Publishing, LLC., 1994.

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<sup>1</sup>Strogatz, 1994

<sup>2</sup>Logan, 1997