An Introduction to Cancer Modeling with Optimal Control

Park City Mathematics Institute
Undergraduate Course, June 27-July15, 2005

Instructor: Lisette de Pillis
Assistant Instructor: Angela Gallegos
Lecture Time: M-F 9:40am – 10:00am
Location: Coalition 1 & 2
Working Session Time: M-F 4:30pm-5:15pm

- **Course Description** The modeling of cancer growth and treatment is one that does not admit only narrow knowledge, but requires skills from multiple disciplines. This field of study lies at the intersection of biology and medicine, with mathematics at the core. Cancer development and the dynamics of the immune system have been a significant focus of mathematical modeling in recent decades. Immunotherapy, a treatment approach that enhances the body's natural ability to fight cancers, is becoming increasingly prevalent in many multi-stage treatment programs that also include chemotherapy, radiation, and surgery. The critical importance of the immune system in combating cancer has been verified both clinically and through mathematical models.

  In this course, we will begin with an overview of the growing field of cancer modeling, surveying the broad number of mathematical techniques that have been taken to attacking this large and complex problem.

  We will then focus on specific models of cancer at the cellular level that include immune system responses, chemotherapy and immunotherapy. Model dynamics will be explored through bifurcation analysis techniques and numerical experiments. We will then introduce the calculus of variations and the mathematical theory of optimal control, which will be applied to these cancer models to determine theoretically improved treatment protocols.

- **Prerequisites** The following background topics are helpful, but if you have not yet been exposed to them, you are still welcome to take the course, and additional assistance will be provided to you as needed. Ideally, the student will have been exposed to the concepts of derivatives and differential equations. It will be helpful if the student has some experience working with mathematical modeling techniques, and is able to carry out a standard phase-plane analysis of a two-dimensional system of non-linear ordinary differential equations. However, all these concepts will be introduced in the course as well. Matlab use will also be introduced, so familiarity is not necessary, but beneficial.

- **Reference Textbooks** The following texts contain information relevant to course content:

  - Steven H. Strogatz, "Nonlinear Dynamics and Chaos"
  - J.D. Murray, "Mathematical Biology" Volumes I and II
  - Leslie M. Hocking, "Optimal Control. An Introduction to the Theory with Applications"
  - Roitt, Brostoff and Male, "Immunology"

- **Coursework**
  - Major Course Project: There will one major course project to be carried out in teams of two. The project involves reading a research paper related to the course topic, and presenting to your peers in the final week the paper as well as your personal insights into the work.

    It is best to create your presentation in PowerPoint, or on overhead transparencies. If there is another medium you wish to use, feel free to discuss your choice with the course instructors.

    Further details of this project will be given out in a separate handout.
  - Afternoon Exercises: In the afternoon, additional mathematical and computational exercises will be given out, and Angela will be helping you to work through the assigned problems.
- **Grading** Your peers will fill out evaluation sheets after viewing your final presentation. These evaluation sheets are meant to serve as constructive feedback.
- **Topics List**

  The following topics will be addressed in this course.
  - Cancer statistics, treatments.
  - The immune system, developing a mathematical model for cancer.
  - Systems of ordinary differential equations, general qualitative analysis approach.
  - Parameter estimation.
  - Nondimensionalization.
  - Bifurcation analysis.
  - Numerical approaches to finding solutions.
  - Calculus of variations, optimal control.
  - Research problems.