Math 171 - Abstract Algebra I
HW # 1
9/6/10

Dummit & Foote 1.1 # 8, 22, 25, 26, 27, 31

1.1.8 Let $G = \{z \in \mathbb{C} \mid z^n = 1 \text{ for some } n \in \mathbb{Z}^+\}$.

(a) Prove that G is a group under multiplication (called the group of *roots of unity* in $\mathbb{C}$).

(b) Prove that G is not a group under addition.

1.1.22 If x and g are elements of the group G, prove that $|x| = |g^{-1}xg|$. Deduce that $|ab| = |ba|$ for all $a, b \in G$.

1.1.25 Prove that if $x^2 = 1$ for all $x \in G$ then G is abelian.

1.1.26 Assume H is a nonempty subset of $(G, \star)$ which is closed under the binary operation on G and is closed under inverses, i.e., for all h and $k \in H$, hk and $h^{-1} \in H$. Prove that H is a group under the operation $\star$ restricted to H (such a subset H is called a *subgroup* of G).

1.1.27 Prove that if x is an element of the group G then $\{x^n \mid n \in \mathbb{Z}\}$ is a subgroup (cf. the preceding exercise) of G (called the *cyclic subgroup* of G generated by x).

1.1.31 Prove that any finite group G of even order contains an element of order 2. [Let $t(G)$ be the set $\{g \in G \mid g \neq g^{-1}\}$. Show that $t(G)$ has an even number of elements and every nonidentity element of $G - f(G)$ has order 2.]