Math 171 - Abstract Algebra I Section __
HW #10
10/11/10

Dummit & Foote (7.2) 3,7,10,13

7.2.3 Let R be a commutative ring with identity and define the set $R[[x]]$ of *formal power series* in x with coefficients from R to be all formal infinite sums

$$\sum_{n=0}^{\infty} a_n x^n = a_0 + a_1 x + a_2 x^2 + \ldots.$$

Recall that addition and multiplication are defined in essentially the same way as for polynomials.

$$\left(\sum_{n=0}^{\infty} a_n x^n\right) + \left(\sum_{n=0}^{\infty} b_n x^n\right) = \sum_{n=0}^{\infty} (a_n + b_n) x^n$$

$$\left(\sum_{n=0}^{\infty} a_n x^n\right) \times \left(\sum_{n=0}^{\infty} b_n x^n\right) = \sum_{n=0}^{\infty} \left(\sum_{k=0}^{n} a_k b_{n-k}\right) x^n$$

(a) Prove that $R[[x]]$ is a commutative ring with identity.

(b) Show that $1 - x$ is a unit in $R[[x]]$ with inverse $1 + x + x^2 + \ldots$.

(c) Prove that $\sum_{n=0}^{\infty} a_n x^n$ is a unit in $R[[x]]$ if and only if a_0 is a unit in R.

7.2.7 The center of a ring R is the set

$$Z(R) = \{r \in R \mid rx = xr \text{ for all } x \in R\}.$$

Let R be a commutative ring with identity. Prove that the center of the ring $M_n(R)$ is the set of scalar matrices, which are scalar multiples of the identity matrix. [Use the elements E_{ij} that we talked about in class.]

7.2.10 Consider the following elements of the integral group ring $\mathbb{Z}S_3$:

$$\alpha = 3(1\,2) - 5(2\,3) + 14(1\,2\,3) \quad \text{and} \quad \beta = 6(1) + 2(2\,3) - 7(1\,3\,2)$$

(where (1) is the identity of S_3). Compute the following elements:

(a) $\alpha + \beta$,

(b) $2\alpha - 3\beta$,

(c) $\alpha\beta$,

(d) $\beta\alpha$,

(e) α^2.

7.2.13 Assume R is a commutative ring with identity. Let $\mathcal{K} = \{k_1, \ldots, k_m\}$ be a conjugacy class in the finite group G.

(a) Prove that the element $K = k_1 + \ldots + k_m$ is in the center of the group ring RG (cf. Exercise 7, Section 1). [Check that $g^{-1}Kg = K$ for all $g \in G$.]

(b) Let $\mathcal{K}_1, \ldots, \mathcal{K}_r$ be the conjugacy classes of G and for each $\mathcal{K}_i$ let K_i be the element of RG that is the sum of the members of $\mathcal{K}_i$. Prove that an element $\alpha \in RG$ is in the center of RG if and only if $\alpha = a_1K_1 + a_2K_2 + \ldots + a_rK_r$ for some $a_1, a_2, \ldots, a_r \in R$.