MATH 171 FALL 2010: HW 19

DUE THU NOV 18

(1) Let H be a group. Define a map

$$\mathcal{H}_\times : \mathcal{G}ps \to \mathcal{G}ps$$

as follows. For all groups G,

$$\mathcal{H}_\times(G) = H \times G,$$

and for the homomorphism $\phi : G \to G'$,

$$\mathcal{H}_\times(\phi) : H \times G \to H \times G'$$

is given by

$$(h, g) \mapsto (h, \phi(g)).$$

Prove $\mathcal{H}_\times$ is a functor.

(2) Show that the map F from $\mathcal{R}ings$ to $\mathcal{G}ps$ defined by mapping each ring to its group of units defines a functor. Moreover, for any two rings R and S, F defines a map

$$F' : \mathcal{H}om(R, S) \to \mathcal{H}om(F(R), F(S)).$$

Here $\mathcal{H}om(A, B)$ is the set of all ring homomorphisms from A to B. Find an example of rings R, S such that F' is not injective. (Such functors are called *not faithful*.)

Date: November 15, 2010.