Math 171 - Abstract Algebra I
HW # 2
9/9/10

Dummit & Foote (1.1) 2, 3. (2.1) 5,6

1.1.2 Decide which of the following binary operations are commutative:

(a) the operation $\star$ on $\mathbb{Z}$ defined by $a \star b = a - b$

(b) the operation $\star$ on $\mathbb{R}$ defined by $a \star b = a + b + ab$

(c) the operation $\star$ on $\mathbb{Q}$ defined by $a \star b = \frac{a+b}{5}$

(d) the operation $\star$ on $\mathbb{Z} \times \mathbb{Z}$ defined by $(a, b) \star (c, d) = (ad + bc, bd)$

(e) the operation $\star$ on $\mathbb{Q} - \{0\}$ defined by $a \star b = \frac{a}{b}$.

1.1.3 Prove that addition of residue classes in $\mathbb{Z}/n\mathbb{Z}$ is associative (you may assume it is well defined).

2.1.5 Prove that G cannot have a subroup H with $|H| = n - 1$, where $n = |G| > 2$.

2.1.6 Let G be an abelian group. Prove that $\{g \in G \mid |g| < \infty\}$ is a subgroup of G (called the *torsion subgroup* of G). Give an explicit example where this set is not a subgroup when G is non-abelian.