

1. Let ABC be an equilateral triangle, and let D be its centroid. The group $G = D_3$ acts on the triangle in the way defined in class. Consider the action this gives on the set $X = \{A, B, C, D\}$.

(a.) What are the orbits of the action of G on X ?

(b.) Find the subgroups G_A and G_D .

(c.) What is X_G ?

(Sometimes called the $n!$ theorem)

2. Let G be a finite group and let $H \leq G$ with $|G : H| = n$.

- a. Show that G acts on the set $\{xH \mid x \in G\}$ by left multiplication.
- b. Show that N , the kernel of this action, is contained in H .
- c. Show that G/N is isomorphic to a subgroup of S_n .
- d. Conclude that $|G : N|$ divides $n!$.

3. Let G be a finite group and let $H \leq G$ with $|G : H| = p$, where p is the smallest prime divisor of $|G|$. Apply the previous problem to show $H \triangleleft G$. (Compare this problem to question 5 from Exam 1.)