

Dummit & Foote (3.1) 1, 6, 7, 9

3.1.1 Let $\varphi : G \rightarrow H$ be a homomorphism and let E be a subgroup of H . Prove that $\varphi^{-1}(E) \leq G$ (i.e., the preimage or pullback of a subgroup under a homomorphism is a subgroup). If $E \trianglelefteq H$ prove that $\varphi^{-1}(E) \trianglelefteq G$. Deduce that $\ker \varphi \trianglelefteq G$.

3.1.6 Define $\varphi : \mathbb{R}^\times \rightarrow \{\pm 1\}$ by letting $\varphi(x)$ be x divided by the absolute value of x . Describe the fibers of φ and prove that φ is a homomorphism.

3.1.7 Define $\pi : \mathbb{R}^2 \rightarrow \mathbb{R}$ by $\pi((x, y)) = x + y$. Prove that π is a surjective homomorphism and describe the kernel and fibers of π geometrically.

3.1.9 Define $\varphi : \mathbb{C}^\times \rightarrow \mathbb{R}^\times$ by $\varphi(a + bi) = a^2 + b^2$. Prove that φ is a homomorphism and find the image of φ . Describe the kernel and the fibers of φ geometrically (as subsets of the plane).