

Dummit & Foote 3.1 # 14, 24, 40, 41

3.1.14 Consider the additive quotient group \mathbb{Q}/\mathbb{Z} .

- (a) Show that every coset of \mathbb{Z} in \mathbb{Q} contains exactly one representative $q \in \mathbb{Q}$ in the range $0 \leq q < 1$.
- (b) Show that every element of \mathbb{Q}/\mathbb{Z} has finite order but that there are elements of arbitrarily large order.
- (c) Show that \mathbb{Q}/\mathbb{Z} is the torsion subgroup of \mathbb{R}/\mathbb{Z} (cf. Exercise 6, Section 2.1).
- (d) Prove that \mathbb{Q}/\mathbb{Z} is isomorphic to the multiplicative group of root of unity in \mathbb{C}^\times .

3.1.24 Prove that if $N \trianglelefteq G$ and H is any subgroup of G then $N \cap H \trianglelefteq H$.

3.1.40 Let G be a group, let N be a normal subgroup of G and let $\bar{G} = G/N$. Prove that \bar{x} and \bar{y} commute in \bar{G} if and only if $x^{-1}y^{-1}xy \in N$. (The element $x^{-1}y^{-1}xy$ is called the *commutator* of x and y and is denoted by $[x, y]$.)

3.1.41 Let G be a group. Prove that $N = \langle x^{-1}y^{-1}xy \mid x, y \in G \rangle$ is a normal subgroup of G and G/N is abelian (N is called the *commutator subgroup* of G).