Math 171 - Abstract Algebra I Section __
HW #9
10/7/10

Dummit & Foote (7.1) 14, 15, 16, 17

7.1.14 Let x be a nilpotent element of the commutative ring R (cf. the preceding exercise).

(a) Prove that x is either zero or a zero divisor.

(b) Prove that rx is nilpotent for all $r \in R$.

(c) Prove that $1 + x$ is a unit in R.

(d) Deduce that the sum of a nilpotent element and a unit is a unit.

7.1.15 A ring R is called a *Boolean ring* if $a^2 = a$ for all $a \in R$. Prove that every Boolean ring is commutative.

7.1.16 Prove that a Boolean ring that is an integral domain has only two elements.

7.1.17 Let R and S be rings. Prove that the direct product $R \times S$ is a ring under componentwise addition and multiplication. Prove that $R \times S$ is commutative if and only if both R and S are commutative. Prove that $R \times S$ has an identity if and only if both R and S have identities.