Imagine Math Day: Encouraging Secondary School Students and Teachers to Engage in Authentic Mathematical Discovery

By Darryl Yong and Michael Orrison

> If we wish to talk about mathematics in a way that includes acts of creativity and understanding, then we must be prepared to adopt a different point of view from the one in most books about mathematics and science. When mathematics is viewed as content, it is lifeless and static...
>
> William Byers, in
> *How Mathematicians Think*

Research mathematicians and school children experience mathematics in profoundly different ways. Ask a group of mathematicians what it means to "do mathematics" and you are likely to get a myriad of responses: mathematics involves analyzing and organizing patterns and relationships, reasoning and drawing conclusions about the world, or creating languages and tools to describe and solve important problems. Students of mathematics often report "doing mathematics" as performing calculations or following rules. It's natural that they see mathematics as monolithic rather than an evolving, growing, socially constructed body of knowledge, because most mathematical training in primary and secondary schools consists of learning how to use pre-existing mathematical tools. They rarely get to see the process by which those tools came about, let alone authentically participate in the construction of those tools.

Of course, this difference in perspectives between research mathematicians and students will always exist, but there are numerous reasons why we should try to bridge the divide. In [2], Cuoco, Goldenberg and Mark argue that the goal of mathematics training should be to help students learn how to think about problems the way mathematicians do. To be able to develop these mathematical "habits of mind," students must be presented with experiences that make it possible for them to authentically create mathematics for themselves.

The need to give children rich and authentic mathematical experiences is also heightened by our current climate of high stakes testing, accountability and standardization. The questions that frequently appear on easily-gradable tests give students a false impression of the kinds of mathematical thinking and skills that are important. Overly prescriptive math curricula limit the opportunities for teachers to engage students in deep mathematical investigations and to convey beautiful but "non-essential" mathematical topics.

Mathematics teachers also need mathematical stimulation, opportunities to remind themselves why teaching, learning, and creating mathematics can be useful, rewarding and fulfilling. They need to be aware of the powerful role that mathematics can play in the lives of their students, not simply because of the mathematical content their students will be asked to master, but also because of the ability of mathematics to be an effective vehicle for teaching students valuable "habits of mind."

Imagine Math Day (IMD) is an outreach activity designed to engage secondary school students and their teachers in authentic mathematical discovery. The Harvey Mudd College (HMC) Professional Development and Outreach Group has designed and carried out this program annually at HMC since 2006.

William Byers has written eloquently in [1] about how ambiguity catalyzes mathematical innovation. In that spirit, the mathematical investigations in each IMD have many valid interpretations and possible "correct" answers. In fact, many of the discussion questions are intentionally designed to increase the likelihood that participants will bring differing, even contradictory, ideas to the conversation; many of the activities and questions lead participants to confusing or surprising "conclusions."

Of course, for this rich and lively mathematical discourse to take place, IMD participants must collaborate with each

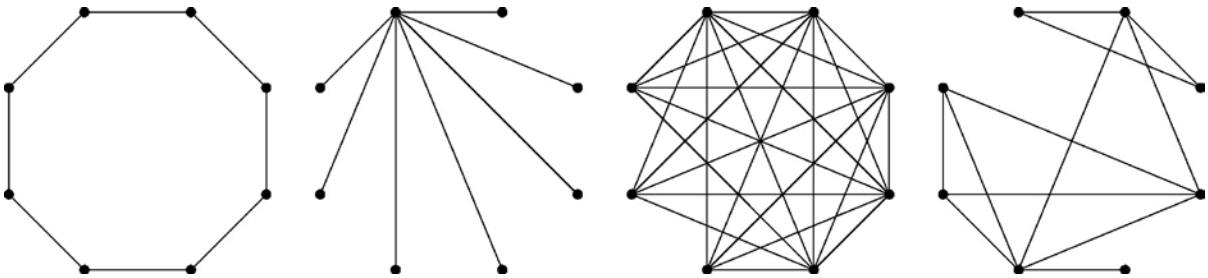

Figure 1: Which of the four graphs above would you say are complex? Can you articulate why one graph is more complex than another? Can you rank these graphs from most complex to least complex?" These intentionally vague questions formed the heart of the 2006 Imagine Math Day activity on graph complexity. There is no "right answer" to these questions, and they led students to create their own notion of the complexity of a graph.

other instead of passively receiving knowledge from others. Each IMD is organized so that no one lectures to the participants. Instead, participants are given a series of discussion questions that lead them deeper and deeper into the mathematical topic for the day.

Another distinctive feature of IMD is that no prizes or awards are given. Even though some of our participants have commented that we should give awards, we felt strongly that because the goal of this activity is to increase interest in and appreciation for mathematics, we wanted to allow *all* participants to feel successful. The atmosphere at IMD is one of sharing and debating ideas, rather than competing to see who can get the answers first.

The spirit of IMD is similar to that of math circles that can be found throughout the world, which also encourage participants to appreciate the beauty of mathematics [8]. Many math circles are also designed to cultivate mathematical talent and some include preparation for mathematical competitions among their activities. In that respect, IMD is different because it is meant to reach students of all dispositions towards mathematics. IMD activities are designed to be unfamiliar to most participants and yet have a very low learning curve so that students who don't identify themselves as "math whizzes" can still participate successfully. Another difference is that while the conversations and ideas that are sparked at IMD may last a long time, the activity itself only lasts one day. We want IMD to have a high impact on participants and to be easily replicable, but we are realistic about the activity's impact on departmental resources and personnel, especially the organizers.

The Logistics of Imagine Math Day

IMD takes place on a Saturday from about 9:30 a.m. to 3:00 p.m. All participants are served breakfast and lunch. There is no cost to participate, but participants are asked to arrange their own transportation to HMC. We send invitations to mathematics teachers and department chairs at public and private high schools around the greater Los Angeles area, and ask teachers to select up to four students to bring with them to the event.

Each of the IMDs so far has been attended by about 80 high school students (grades 9 through 12) and 20 teachers from the greater Los Angeles area. The size limit is an artificial one, imposed by the room at HMC that we use for the activity, but we suspect that any more than this number of participants would make the activity feel less intimate and too formal.

As mentioned earlier, one key to the success of Imagine Math Day has been to foster rich mathematical inquiry and dialog between students and teachers. We choose topics that are likely to be unfamiliar to both students and teachers, so that all of the participants engage in authentic mathematical discovery at the same time. Participants sit at tables of ten: on average, eight students and two teachers. We have found that this number allows for students to pick up on teachers' enthusiasm, more articulate mathematical thinking, and problem solving skills. More importantly, students see themselves as working with their teachers rather than for their teachers.

The only time one person speaks to the entire group is during the brief welcome at the beginning of the day. We encourage participants to conjecture, explore, and hypothesize with those around them while being respectful of others' ideas; we make it very clear that IMD is not a competition and that everyone is encouraged to participate. For the rest of the morning, all of the participants work through a series of carefully planned questions and discussion prompts.

During lunch, HMC math majors and faculty eat and mingle with the participants and ask them about their experiences during the morning and also talk to them about studying mathematics in college. The student participants are then organized into small groups and charged with further exploring something related to the day's activities and creating a poster summarizing their findings. During this time, the teachers are invited to talk with each other and hear more about the mathematical background on the topic for the day. At the end of the day, students put up their posters, creating a gallery of different mathematical ideas.

Figure 2: A group of pre-service teachers grapples with how to measure angles on a realization of the hyperbolic plane.

The Mathematics Behind Imagine Math Day

The main key to the success of Imagine Math Day has been the choice of highly accessible, stimulating, and engaging topics. These topics are introduced through a series of worksheets that give participants just enough

information to spark their interest and allow them to carry on conversations at their tables without limiting their creativity.

In 2006, the topic was graph complexity. After a brief introduction to graph theory, students and teachers engaged in one question — what makes a graph more complex than another? In 2007, the topic was non-Euclidean geometries. Participants explored how familiar ideas from Euclidean geometry become unfamiliar, vague, and thought-provoking when we try to conceive them on spheres, footballs, or other surfaces.

This year, the topic was the mathematics of voting, an especially appropriate topic since it is an election year. After exploring how the choice of different voting procedures can have a dramatic impact on the outcome of an election, participants were given the task of creating their own voting paradoxes. During the afternoon, they were then given the open-ended task of

Figure 3: Participants of the 2008 IMD think about alternative systems for presidential elections.

investigating a voting scheme or paradox. For example, one of the possible activities was to design "the next popular reality TV show with an interesting voting system."

Evaluating the Success of IMD from Participant Responses

At the end of each IMD, all participants (students and teachers) are asked to anonymously submit written comments about their experiences. Student comments often suggest that their view of mathematics has grown. "Math is usually considered something with one right answer," said one student, "but [I see now that] there is room for interpretation sometimes." Another commented that "The activity showed me I could have fun while doing mathematical problems."

Teachers also responded positively. One said that the activity "reminded me how enjoyable math can be for students," and pointed out how easy it is to not take the time to let students

Figure 4: A student discusses the ideas behind her group's alternative voting system to her teacher.

"live the math." Another told us that two of the students who had been to Imagine Math Day the previous year "begged to go again" despite the fact that they were not taking a mathematics course.

Some comments reflected students' fairly narrow view of mathematics. One student said that the activity on graph complexity did not change his view of mathematics "because it technically is not related to math." Another asked us to lecture more, and one commented that the activity on voting "really had no mathematics" in it.

Though these comments sound negative, they hint at IMD's success in challenging students' conceptions about doing and learning mathematics. A mathematical problem is fruitful when it leads to more questions than answers and in the same way, we think that IMD has been successful when it raises questions for participants rather than gives them the correct answers. Our hope is that participation in IMD will lead to seeds that might bloom into conversations between students and teachers about mathematical ideas and what it means to do mathematics. Through conversations like these, the gap between research mathematicians and students of mathematics might be narrowed.

Imagining IMD on Your Campus

We are confident that IMD can be easily replicated (and modified if necessary) in a variety of settings, and we are eager to share our insights and experiences with others. To that end, all of the materials created for previous IMDs are available at http://www.math.hmc.edu/pdo/imd/.

We would also enjoy learning from the experiences of others who are interested in similar outreach activities. In particular, we are always on the lookout for mathematical topics that are

accessible and inviting to secondary mathematics students, yet rich enough to encourage genuine exploration, thought, and even wonder. By meeting mathematics at its broadest, more students may recognize their aptitude for it. By working together, we can make access to high quality mathematical experiences available to all students.

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