

Harvey Mudd College Math Tutorial:

Algebra Review

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Here we will list some of the most common mistakes that we see on exams. If you can avoid these, then at least your mistakes will be uncommon. Most of the mistakes that occur repeatedly involve algebra, rather than calculus. They can be avoided by being careful and checking your work. Others involve common misunderstandings about various aspects of calculus.

1. $(x + y)^2 = x^2 + y^2$. **MISTAKE!**

Powers don't behave this way. The correct way to expand this expression gives

$$(x + y)^2 = x^2 + 2xy + y^2 < /sup >$$

2. $\frac{1}{x + y} = \frac{1}{x} + \frac{1}{y}$. **MISTAKE!**

The rule for adding fractions gives

$$\frac{1}{x} + \frac{1}{y} = \frac{x + y}{xy}$$

3. $\frac{1}{x + y} = \frac{1}{x} + y$. **MISTAKE!**

This very common error comes from carelessness about what's in the denominator. Can be avoided by careful handwriting or frequent use of parentheses.

4. $\sqrt{x + y} = \sqrt{x} + \sqrt{y}$. **MISTAKE!**

There is no simplified way to write $\sqrt{x + y}$. You just have to live with it as is.

5. $x < y$ so $kx < ky$ where k is a constant. **MISTAKE!**

This is true when k is a POSITIVE constant. If k is negative you need to reverse the inequality. If k is zero all bets are off. For example, if $x < y$ then $-x > -y$.

6. Forgetting to simplify fractions in limits: **MISTAKE!**

It is not correct to say $\lim_{x \rightarrow 1} \frac{x^2 - 1}{x - 1} = \frac{0}{0}$ and therefore the limit is undefined. Even worse would be to cancel the zeroes and say the limit equals one.

Any time you get $\frac{0}{0}$ for a limit, it is a BIG WARNING SIGN that says YOU HAVE MORE WORK TO DO! In this case,

$$\lim_{x \rightarrow 1} \frac{x^2 - 1}{x - 1} = \lim_{x \rightarrow 1} \frac{(x + 1)(x - 1)}{x - 1} = \lim_{x \rightarrow 1} x + 1 = 2$$

7. $\sin 2x/x = \sin 2$. **MISTAKE!**

You can only cancel terms in the numerator and denominator of a fraction if they are not inside anything else and are just multiplying the rest of the numerator and denominator. The function $\sin(2x)$ is NOT $\sin 2$ multiplied by x . If the fraction had been written as

$$\frac{\sin(2x)}{x}$$

it would be harder to make such an error.

8. $ax = bx$ therefore $a = b$. **MISTAKE!**

This is a more subtle mistake. The cancellation is correct IF x is not 0. For example $2x = 3x$ forces $x = 0$. You cannot cancel the x and conclude that $2 = 3$. Not in this universe, anyway.

9. $\frac{d}{dx}[2^x] = x2^{x-1}$. **MISTAKE!**

The correct answer is $2^x \ln x$. The power rule only applies if the base is a variable and the exponent is a constant, as in x^3 .

10. $\frac{d}{dx}[\sin(x^2 + 1)] = \cos(2x)$. **MISTAKE!**

This is a typical example of the kind of mistakes made when applying the chain rule. The correct answer is

$$\frac{d}{dx}[\sin(x^2 + 1)] = \cos(x^2 + 1) \times 2x$$

11. $\frac{d}{dx}[\sin(x^2 + 1)] = \cos(x^2 + 1) + \sin(2x)$. **MISTAKE!**

Another common way in which the chain rule is misapplied. This time the product rule has been used in a setting where the chain rule was the way to go.

12. $\frac{d}{dx}[\cos x] = \sin x$. **MISTAKE!**

The answer should be $-\sin x$. Extremely common error costing students over 10 million points a year on exams around the world.

13. $\frac{d}{dx}\left[\frac{f}{g}\right] = \frac{fg' - gf'}{g^2}$. **MISTAKE!**

This is backwards! It should be

$$\frac{d}{dx}\left[\frac{f}{g}\right] = \frac{gf' - fg'}{g^2}$$

14. $\frac{d}{dx}[\ln 3] = \frac{1}{3}$. **MISTAKE!**

The quantity $\ln 3$ is a constant, so $\frac{d}{dx}[\ln 3] = 0$. The same is true for ALL constants. So $\frac{d}{dx}[e] = 0$ and $\frac{d}{dx}\left[\sin\left(\frac{\pi}{2}\right)\right] = 0$ as well.

15. $\int x, dx = \frac{x^2}{2}$. **MISTAKE!**

The correct answer is $\int x dx = \frac{x^2}{2} + C$. Picky profs penalize points pedantically.

16. $\int \frac{1}{x} dx = \frac{x^0}{0} + C$. **MISTAKE!**

The power rule for integration does not apply to x^{-1} . Instead,

$$\int \frac{1}{x} dx = \ln|x| + C$$

17. $\int \tan x dx = \sec^2 x + C$. **MISTAKE!**

It's the other way around. $\frac{d}{dx}[\tan x] = \sec^2 x$. The correct answer is

$$\int \tan x dx = \ln|\sec x| + C$$

as can be found by u -substitution with $u = \cos x$.

18. Forgetting to simplify: **MISTAKE!**

For example

$$\int x\sqrt{x} dx$$

is easy if you notice that $x\sqrt{x} = x^{3/2}$ and then apply the power rule for integration. But if you try to do it using integration by parts or substitution, you will find yourself in outer space without a space suit.

19. Not substituting back to the original variable: **MISTAKE!**

$$\int 2xe^{x^2}$$

does not equal $e^u + C$. It equals $e^{x^2} + C$.

20. Misreading the problem: **MISTAKE!**

If asked to find an area, don't find a volume. If asked to find a derivative, don't find an integral. If asked to use calculus to solve a problem, don't do it in your head using algebra. Although it seems silly to include this item in our list, billions of points have been taken off exams for mistakes of this type. After you finish a problem on the exam, go back and read the question again. Check to make sure you answered the question that was asked.

21. Thinking you're prepared when you're not. **MISTAKE!**

This mistake is perhaps the most important, so we'll put it in even though it pushes us over the 20 mistakes limit. The worst mistake many students make is to think they know the material better than they really do. It's easy to fool yourself into thinking you can solve a problem when you're looking at the answer book or at a worked out solution. Test your knowledge by trying problems under exam conditions. If you can do them under that restriction, the exam should be a breeze.