MATH 14 MIDTERM REVIEW

WARNING: The following problems do not cover all the material covered in class. You MUST go over your notes and old homework in order to obtain a complete review.

1. Circle **T** for true, or **F** for false for each question. You do NOT have to show your work.

   T F (a) If a function $f : \mathbb{R}^n \longrightarrow \mathbb{R}$ is continuous at $\vec{x} = \vec{a}$, then f must have a limit as $\vec{x}$ approaches $\vec{a}$.

   T F (b) If $f : \mathbb{R}^2 \longrightarrow \mathbb{R}$ is differentiable at $\vec{x} = \vec{a}$, then both partial derivatives, $\frac{\partial f}{\partial x}$, $\frac{\partial f}{\partial y}$ exist at $\vec{x} = \vec{a}$.

   T F (c) If $f : \mathbb{R}^2 \longrightarrow \mathbb{R}$, and both partials, $\frac{\partial f}{\partial x}$, $\frac{\partial f}{\partial y}$ exist at $\vec{x} = \vec{a}$, then f is differentiable at $\vec{x} = \vec{a}$.

   T F (d) Suppose $f : \mathbb{R}^2 \longrightarrow \mathbb{R}$ is a function. If the level curve for $c = 1$ is the line $x = 1$ (minus the point (1,1)) and the level curve for $c = 2$ is the line $y = x$ (minus the point (1,1)), then f has no limit as $(x, y) \longrightarrow (1, 1)$.

2. Find an equation of the plane that passes through the points $(1, 5, 1), (3, 1, 3)$, and $(-3, 4, 3)$. Your answer should be in the form $Ax + By + Cz = D$.

3. Compute each limit or show that the limit does not exist:

   (a) $\lim_{(x,y)\to(0,0)} 14e^{-x^2-y^2}$

   (b) $\lim_{(x,y)\to(0,0)}(\frac{x^{12}-y^{12}}{x^4+y^4} + 14)$

(c) $\lim_{(x,y)\to(0,0)} \frac{84e^{x^2+y^2}-84-84x^2-84y^2-42(x^2+y^2)^2}{(x^2+y^2)^3}$

(d) $\lim_{(x,y)\to(0,0)}(x\sin(\frac{x}{x^2+y^2})+14)$

4. Find an equation of the tangent plane to the surface at the specified point.

   (a) $z = x^{14} + y^{14}$ at the point $(1, 1, 2)$.

   (b) $x^{14} + y^{14} + z^{14} = 3$ at the point $(1, 1, 1)$.

5. Let $f : \mathbb{R}^2 \longrightarrow \mathbb{R}^3, f(x, y) = (x^{14}y^{14}, \frac{1}{2}x^{28}y^{28}, 14x + 14y)$. Compute $Df(1, 1)$.

6. Compute the directional derivative of the function $T(x, y) = x^3 - y^2$ at the point $(1, 2)$ in the direction $(5, 12)$.

7. A function $f(x, y)$ is said to be harmonic if it satisfies Laplace's equation: $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0$. Two functions u and v are said to satisfy the Cauchy-Riemann equations if $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$ and $\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$. Prove that if u and v satisfy the Cauchy-Riemann equations, then u and v are harmonic.