Math 12 Linear Algebra I Sample Exam Questions

1. a) Determine the eigenvalues of the matrix

$$A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 0 & 2 \\ 0 & 1 & 1 \end{bmatrix}$$

b) Compute the inverse of A.

2. For the matrix

$$A = \begin{bmatrix} 3 & 0 & 2 & 0 \\ 1 & 3 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix}$$

a) Find a basis for the eigenspace of $\lambda = 4$.

b) Verify that $\begin{pmatrix} -2 \\ 1 \\ 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 0 \\ -1 \\ 0 \end{pmatrix}$ are eigenvectors of A.

c) Compute the determinant of A.

3. Let $a_1 = \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix}$, $a_2 = \begin{pmatrix} 1 \\ 1 \\ -1 \\ 0 \end{pmatrix}$, $v_3 = \begin{pmatrix} 2 \\ 2 \\ 3 \\ 0 \end{pmatrix}$. Are these vectors linearly independent?

4. Prove the following statement. If vectors x, y, and z are linearly independent, then the vectors x, $x + y$, $x + y + z$ are linearly independent.

5. A square matrix A is called *skew-symmetric* if $A^T = -A$. Prove the set of n-by-n skew-symmetric matrices is a subspace of all n-by-n matrices.