Math 62 Statistics Sample Exam Questions

1. (10) Explain the difference between the distribution of a population and the sampling distribution of a statistic, such as the mean, of a sample randomly selected from that population.

2. (10) Select a sample of size $n = 10$ from a normal population. For what value t_0 will $\Pr[t < t_0] = 0.01$?

$$t_0 = \boxed{}$$

Show your work:

3. (10) The duration of Alzheimer's disease ranges from 3 to 20 years. The average is 8 years and the standard deviation is 4 years. For a clinical study 30 patients are randomly selected that have been determined to be at the very beginning stage of the disease.

   (a) What is the probability that the average duration of the disease among the sampled patients will be less than 7 years?

   Probability = $\boxed{}$

   (b) What is the probability that the average duration of the disease among the sampled patients will be between 7 and 9 years?

   Probability = $\boxed{}$

4. (10) A random sample of 985 likely voters was taken. 592 indicated that they approved of a reduction in the federal income tax. Construct a 95% confidence interval for the true proportion p of voters who approve of reducing the income tax.

$$\boxed{} < p < \boxed{}$$

Show your work:

5. (15) On the first E & M test, we compared the performance of Pilot Math students to Non-Pilot Math students who had been in Math 3 last year. The following data was collected:

GROUP	n	$\bar{x}$	s
Pilot	54	58	15
Non-Pilot	30	53	13

Test the hypothesis that these two means are the same. Fill in the blanks, and show your scratch work on this page.

(a) List the assumptions that you must make to perform the test.

(b) Compute the relevant test statistic and its p-value for the two-sided test of the hypothesis that the difference of means is zero:

$z =$ _____, p-value = _____

(c) Do you accept or reject the hypothesis at the 95% significance level?

6. (15) One might hypothesize that performance on math exams will be better in the 9:00 section than the 8:00 section because more students attend the later section and are (presumably) more awake. The following data presents the cumulative performance on the first three exams:

GROUP	n	$\bar{x}$	s
8:00	23	223	34
9:00	30	238	35

Test the hypothesis that these two means are the same. Fill in the blanks, and show your scratch work on this page.

(a) List the assumptions that you must make to perform the test and assess their plausibility.

(b) Compute the relevant test statistic and its p-value for the one-sided test of the hypothesis that the difference of means is zero, against the alternative that the mean at 9:00 is greater than that at 8:00:

Test statistic = $\boxed{}$, p-value = $\boxed{}$

(c) Do you accept or reject the hypothesis at the 95% significance level?

7. (15) A random sample of $n = 12$ observations from a normal population produced the following estimates: $\bar{x} = 47.1$ and $s^2 = 4.7$.

   (a) Test the hypothesis $H_0 : \mu = 48$ versus $H_a : \mu \neq 48$ with $\alpha = 0.1$.

   (b) What is the p-value for your test? p-value = $\boxed{}$

   (c) Find a 90% confidence interval for the mean, and interpret this interval.

$$\boxed{} < \mu < \boxed{}$$

8. (10) A manufacturer of concrete claims that his product has a reasonably stable compressive strength, measured in units of kilograms per square centimeter. He reports that the range of strength observed over many batches of concrete is 40 kg/cm^2. Suppose that you are considering purchasing concrete from this manufacturer, but you must be convinced that the variability in strength is no greater than what is claimed. Your engineers take a random sample of $n = 10$ batches of the material and test their strength, giving sample estimates of $\bar{x} = 312$ and $s^2 = 195$. If you assume that the range of a normal distribution is about 4σ, test the hypothesis that the variance in strength of the sampled concrete is consistent with the manufacturers claims. Use $\alpha = 0.05$ in your test.

   (a) What is the appropriate test statistic to use, with how many degrees of freedom?

   Test statistic = [ ], df = [ ]

   (b) Your null hypothesis is H_0 : [ ] versus H_a : [ ]

   (c) What is the value of your test statistic = [ ]

   (d) Critical value of test statistic ($\alpha = 0.05$) = [ ]

   (e) Do you accept or reject the hypothesis?

9. (15) The following set of data was collected to determine the effects of sleep deprivation on students' ability to solve problems. The amount of sleep deprivation varied, with 8, 12, 16, 20 and 24 hours without sleep. A total of ten subjects participated in the study. A set of simple addition problems was administered to each subject after his or her sleep deprivation period, and the number of errors recorded. These results were obtained:

Number of Errors	8	6	6	10	8	14	14	12	16	12
Hours without Sleep	8	8	12	12	16	16	20	20	24	24

Perform a linear regression and analysis of variance for these data and fill in the blank spaces:

(a) Your formula for the regression line: $\hat{y} =$ [ ] $+$ [ ] x

(b)

Analysis of Variance			
Source	DF	SS	MS
Regression			
Error			
Total		112.4	•

(c) The coefficient of determination is: $r^2 =$ [ ]

(d) Determine a 95% confidence interval for the slope β of the linear model:

[ ] $< \beta <$ [ ]

(e) Compute the relevant t-statistic and p-value for the two-sided test of the hypothesis that $\beta = 0$:

$t =$ [ ], df $=$ [ ], p-value $=$ [ ]

(f) Do you think these data support the thesis that sleep deprivation leads to decreased mental acuity?