

# Rates of Convergence to Self-Similar Solutions of Burgers' Equation

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Burgers' Equation  $u_t + cuu_x = \nu u_{xx}$  is a nonlinear partial differential equation which arises in models of traffic and fluid flow. It is perhaps the simplest equation describing waves under the influence of diffusion. We consider the large-time behavior of solutions with exponentially localized initial conditions, analyzing the rate of convergence to a known self-similar single-hump solution. We use the Cole-Hopf Transformation to convert the problem into a heat equation problem with exponentially localized initial conditions. The solution to this problem converges to a Gaussian. We then find an optimal Gaussian approximation which is accurate to order  $t^{-2}$ . Transforming back to Burgers' Equation yields a solution accurate to order  $t^{-2}$ .