Research Proposal:
Fractional Analogues in Graph Theory

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The field of fractional graph theory is based upon the premise of replacing the set 0,1 with [0,1] when specifying membership sets. When a "fuzzy set" is defined in this way, integer-valued concepts in the field of graph theory can take on nonintegral values. There are two approaches to this conversion. First, one can define quantities in terms of integer programs and use the linear relaxation of the program to represent the fractional version of the quantity. Second, the resulting limit of the subadditivity lemma can be used to define these quantities. Fortunately, these two approaches often yield identical results. In my senior research, I intend to examine a variety of open problems in the field of fractional graph theory.

I. Prior Work

I have taken Math 108: Graph Theory from Professor Levin and am currently reading Edward R. Scheinerman and Daniel H. Ullman, *Fractional Graph Theory*.

II. Plans For Research

As fractional graph theory is a relatively young field, there are several open questions which I would like to investigate.

The four-color theorem has long been a thorn in the side of mathematicians. Is it possible to prove the fractional version without relying on the original theorem? Tait's theorem gives a specific classification of four-colorable planar graphs, and it is not unthinkable that, using this information, there exists a proof that fractional chromatic number has a maximum of four for all planar graphs.

The chromatic number of the plane, $\chi(\mathbb{R}^2)$, is easily proven to be between four and seven, inclusive. No better bounds have been shown for this quantity. However, it is known that $\chi_f(\mathbb{R}^2)$, the fractional chromatic number of the plane, is between 3.555 and 4.36. Can these bound be improved?

In normal graph theory, the chromatic number $\chi(G)$ is bounded below by the clique number, $\omega(G)$. For finite graphs, the strong duality of linear programming shows that their fractional analogs are equal: $\chi_f(G) = \omega_f(G)$. However, this result is not true for infinite graphs. It is possible with the Mycielski construction to create a graph with any values of $\omega(G)$ and $\chi(G)$ such that $\omega(G) \leq \chi(G)$. Is it possible in the fractional case?

A final topic I plan to study is fractional dimension of posets. Although I do not have specific problems in mind to explore, it will be challenging to gain some in-depth understanding of this subject.