Research Proposal:
Computational Simulation of Fluid Dynamics in Thin Films

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This problem will involve writing a computer code to generate numerical solutions to the standard lubrication equation with surface tension,

$$\frac{\partial h}{\partial t} + \frac{\partial}{\partial x}\left(C\frac{h^3}{3}\frac{\partial^3 h}{\partial x^3} + f\left(h, h_x, h_{xx}\right)\right) = 0,$$

where $h(x,t)$ is the height of a fluid film, for fluid films on surfaces. I will modify the equation to model fluids on surfaces tilted away from the horizontal by various angles, then look for self-similarity in the solutions. In other words, I will try to express the solutions at different angles in inclination as a function of some single scaling parameter.

I have taken Numerical Analysis, in which I learned quite a bit about solving DE's numerically; Dynamical Systems, which dealt with analysis of properties of systems such as self-similarity; and Statistical Mechanics, which dealt with the mechanics of systems of large numbers of particles but not fluid mechanics. I am in the process of reading portions of Viscous Flow by H. and J. R. Ockendon[1], "Thin films with High Surface Tension" by T. G. Myers[2], and "Stability of Self-Similar Solutions for van der Waals driven thin film rupture" by Thomas Witelski and Prof. Bernoff[3]. Over the summer, I plan to read "Long-scale evolution of thin liquid films" by Alexander Oron et al.[4] I also plan to learn C.