Research Proposal:
Harmonic Numbers

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For my senior thesis, I would like to continue working on research to be done this summer with Professor Benjamin. We will be looking for a closed form combinatoric representation of the harmonic numbers. The harmonic number H_n is the n$^{\text{th}}$ partial sum of the harmonic series:

$$H_n = 1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{n}$$

The second order harmonic numbers are partial sums of the normal harmonic numbers:

$$H_n^{(2)} = H_1 + H_2 + \ldots + H_n$$

And we can define the k$^{\text{th}}$ order harmonic number as the partial sum of the $k-1$ order harmonic numbers:

$$H_n^{(k)} = \sum_{i=1}^{n} H_i^{(k-1)}$$

We seek to discover a combinatorial formula for k$^{\text{th}}$ order harmonic numbers. It is not guaranteed that such a formula exists, but we believe it does due to known identities such as:

$$H_n^{(k)} = \binom{n+k+1}{k-1} \left(H_{n+k-1} - H_{k-1}\right)$$

If a suitable formula is found, then we wish to come up with an intuitive physical model for harmonic numbers from the formula, which will give insight and be useful in proving known identities and discovering new ones.

Some preliminary reading has been done in The Book of Numbers, Proofs and Confirmations, The Theory of Partitions, Fibonacci & Lucas Numbers, and the Golden Section, and in Mathematics Magazine.