Mathematics Thesis Proposal

Dispersion of a Passive Scalar in Helical Pipe Flow

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Project Description

This thesis will investigate the advection and diffusion of a passive scalar in helical pipes. By assuming that the curvature and torsion are small (equivalent to small Dean number) and the Reynolds number is moderate, one can use a closed form approximation, due to Dean (1927) and Germano (1982), for the induced recirculation. Using a simulation developed this summer, the problem will be investigated numerically with a range of localized initial conditions.

Already, we have found that the problem is governed by two parameters: a nondimensional diffusion constant D (typically small), and the scaled ratio of torsion to curvature λ. At small times, the longitudinal width of the particle distribution, σ, is governed by diffusive effects ($\sigma \propto \sqrt{Dt}$). At large times, Taylor diffusion dominates ($\sigma \propto \sqrt{t/D}$). However, at intermediate times, a ballistic region exists where the width spreads linearly, as postulated by Mezic & Wiggins (1994).

This project has two parts. First, we will investigate numerical solutions to provide insight into the behavior of the helical pipe flow. Second, we will analyze the system of PDEs generating the flow to explain the various width-scaling regions and to find how the various diffusion regimes scale with D and λ.

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Second Reader: Anette Hosoi

References:

Aref, H. 1989 Chaotic Advection by Laminar Flow in a Twisted Pipe. *J. Fluid Mech.*, 335-357.

Dean, W. R. 1927 Note on the motion of fluid in a curved pipe. *Phil. Mag.* **5**, 673-693.

Germano, M. 1982 On the effect of torsion on a helical pipe flow. *J. Fluid Mech.*, 1-8.

Latini, M. & Bernoff, A. J. 2001 Transient Anomalous Diffusion in Poiseuille Flow. Under consideration for publication in *J. Fluid Mech.*

Mezic, Igor & Wiggins, Stephen. 1994 On the origin of asymptotic t^2 dispersion of a non-diffusive tracer in incompressible laminar flows. *Phys. Fluids* **6**, 2227-2229.

Relevant Course Work:

Math 136, 180, 182
Computer Science 60, 70