Tucker's Labeling in Branched Spheres and Connections to Topology

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There are many connections between combinatorics and topology — famous results in topology such as the Brouwer Fixed Point Theorem and Borsuk-Ulam Theorem have combinatorial analogues that often have constructive proofs, unlike their counterparts in topology. This allows one to find solutions to related problems, such as finding fixed points of highly nonlinear maps or solving certain "fair division" problems in game theory.

In my senior thesis, I propose to investigate a generalization of a result known as Tucker's combinatorial theorem, which constructively shows the existence of a certain kind of feature in labelings of a triangulated sphere. A generalization of this is a conjecture concerning labelings of branched spheres which, if proved, would imply a number of different results in analysis and topology. For instance, given n measures on a set, is there a partition into k pieces so that the n measures on each piece are all equal to $1/k$? The existence has been shown by Lyapounov's theorem (1946), but the proof was not constructive. Proving the generalization of Tucker's theorem may lead to a constructive version of Lyapounov's theorem.

I have done little work on the subject as of yet, although course work in topology and probability has given me a strong background in the areas that will be required for this research. While working on this project, I hope to learn numerous combinatorial and topological arguments and how to apply them to other common problems. I also hope to learn more ways to approach higher dimensional problems, as these problems continually occur in higher mathematics.