

Research Proposal: Results on the Hausdorff Dimension of Julia Sets

Stephen E. Haas

Faculty Advisor: Prof. Lesley A. Ward

1 Introduction

I have greatly enjoyed my previous experiences in complex dynamics. Thus, I have decided to do research in this area.

2 Proposed Research

I hope to study the Hausdorff Dimension of the Julia sets of complex functions. There are a number of open questions in this subject; for instance, does the Hausdorff dimension of the Julia set of a quadratic function of the form $f(z) = z^2 + c$ vary continuously with the parameter c ? While this question is likely beyond the scope of a senior thesis, Professor Ward and I are confident that we can identify a suitable subproblem or easier related problem for my project. For instance, we might be able to study continuity in some portion of the parameter space or for another simple class of functions.

If the area above does not yield a suitable problem we will investigate other areas of complex analysis, such as the local connectivity of the Mandelbrot set. If complex dynamics proves intractable as a thesis topic we will look to other subfields of complex analysis for a topic.

3 Prior Research

My preparation begins with two courses in complex analysis, two courses in real analysis, one course in complex dynamics, and one course in topology; this gives me a strong background in the tools I will need to investigate this subject. Over the past summer I have read from Beardon [1] to learn some of the specific mathematics needed, including how to calculate the Hausdorff dimension for simple examples, such as the Cantor set, the von Koch snowflake, and the Sierpinski carpet and gasket.

I have begun a literature search to find a precise question to investigate. This has included articles such as [4], which gives a detailed discussion on continuity of the Hausdorff dimension of Julia sets for a significant set of simple functions. I am hopeful that this will lead to a research topic soon.

References

- [1] A. F. Beardon *Iteration of Rational Functions*
- [2] Douady, Adrien. *Does a Julia set depend continuously on the polynomial?* Complex Dynamical Systems (Cincinnati, OH, 1994), 91-138, Proceedings of Symposia in Applied Mathematics. American Mathematical Society, Providence, RI, 1994.
- [3] Jenkinson, Oliver; Pollicott, Mark. *Calculating Hausdorff dimensions of Julia sets and Kleinian limit sets.* American Journal of Mathematics 124(2002), no. 3, 495-545.
- [4] Rivera-Letier, Juan. *On the continuity of Hausdorff dimension of Julia sets and similarity between the Mandelbrot and Julia sets.* Fundamenta Mathematicae 170(2001) no.3, 287-317.
- [5] Urbanski, Mariusz and Zinsmeister, Michel. *Continuity of Hausdorff dimension of Julia-Lavaurs sets as a function of the phase.* Conformal Geometry and Dynamics. 5(2001) 140-152.