Thesis Proposal: Separating Sets for Finite Groups

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Abstract

For my thesis, I will investigate separating sets for finite groups. The goal will be to research methods for finding minimal separating sets for a finite group.

1 Prior Work

My prior work pertaining to this project consists of study in the areas of Abstract Algebra and Linear Algebra. I have taken Math 171 (Abstract Algebra I), and will have completed Math 173 (Advanced Linear Algebra) by the end of this semester at Harvey Mudd College. I have taken a course in Galois Theory and will have completed some research in algebra during the Trinity REU by the end of this summer.

2 Intended Reading

Over the next few months, I will read several sections of [1] that are relevant to the upcoming research, working problems and carefully studying the definitions and theorems that are pertinent to my thesis. In addition, there are several relevant papers that I will be reading carefully; among these are [2], [3], [4], and [5].

3 Original Research

Let $V = V_1 \oplus V_2 \oplus \ ... \ \oplus V_n$ be a representation of a finite group where the V_i's are invariant subspaces of V. We want to compute the projections of a vector $v \in V$ into these subspaces. A separating set of V with respect to the decomposition is a collection $\{T_1, ...T_l\}$ of simultaneously diagonalizable linear transformations of V such that the eigenspaces of the T_i are direct sums of the V_i and if $V_i \neq V_j$, then we can find some T_k that assigns distinct eigenvalues to V_i and V_j. In other words, we can distinguish the V_i with this separating set.

If the V_i are subspaces of V known as isotypic subspaces, one well-known method of finding separating sets of a finite group G is to use class sums (see [2]). A class sum is defined to be the element of the group ring $\mathbb{C}G$ that is the sum of all elements in a fixed conjugacy class of G. It is well-known that the collection of all of the class sums forms a separating set. However, this can often be done with far fewer class sums. For example, in the case of S_n, $n \leq 5$, the class sum of the transpositions is sufficient to distinguish distinct isotypic subspaces. But this single class sum does not suffice for S_6. It would be interesting to know how many class sums are needed for other finite groups such as the alternating group, the dihedral group, or wreath products. In particular, it would be interesting to know which class sums should be selected. If k class sums are needed, this does not necessarily imply that any k class sums will do.

Finding a minimal separating set of a representation of a finite group is useful because then we can use eigenspace projection techniques to compute projections onto invariant subspaces of the representation [2]. Separating sets have emerging applications to signal processing and image processing, for which it is very useful to have as small a separating set of a representation V as possible.

The goal of this research will be initially to examine how to use class sums to obtain minimal separating sets of a finite group, and to discover how few class sums are needed to accomplish this. An additional possible direction of research would be to find out if there are any other efficient ways of finding minimal separating sets.

References

[1] Dummit, David and Foote, Richard. *Abstract Algebra, 2nd Edition.* John Wiley & Sons, 1999.

[2] Maslen, David K., Orrison, Michael E., Rockmore, Daniel N. Computing isotypic projections with the Lanczos iteration.

[3] Katriel, J. Some useful results concerning the representation theory of the symmetric group. *J. Phys. A: Math. Gen. 24* 5227-5234, 1991.

[4] Orrison, Michael E. An eigenspace approach to decomposing representations of finite groups.

[5] Orrison, Michael E. Radon transforms and the finite general linear groups.