References

[1] William A. Adkins and Steven H. Weintraub. *Algebra: An Approach via Module Theory*. Number 136 in Graduate Texts in Mathematics. Springer, New York, 1992.

> ANNOTATION: This text presents the material for a first-year graduate course in algebra from a module-theoretic viewpoint. Covers groups, rings, and fields, modules and their structure theorems, and some linear algebra, as well as additional topics in module theory and the representation theory of finite groups. Especially useful as a reference on module theory and module-theoretic formulations of representation theory.

[2] Michael Clausen. Fast generalized fourier transforms. *Theoretical Computer Science*, 67:55–63, 1989.

> ANNOTATION: Clausen improves the upper bound for the linear complexity of FFTs on a general group G using bases adapted to chains of subgroups. The results also apply to inverse FFTs. He then derives an $O(n^3|S_n|)$ upper bound for FFTs on the symmetric group S_n.

[3] Michael Clausen. Elements of a general algebraic theory of standard tableaux. In A. Betten, A. Kohnert, R. Laue, and A. Wassermann, editors, *Algebraic Combinatorics and Applications*, pages 67–78, Berlin, 2001. Springer-Verlag.

> ANNOTATION: Clausen generalizes the standard Young tableaux used to determine the irreducible representations of the symmetric group to tableaux that apply to groups that have multiplicity-free character graphs (MC-groups). This framework is applied to supersolvable groups to yield an efficient determination of their irreducible representations. Such a decomposition is a first step towards determining FFTs on such MC-groups.

[4] Michael Clausen and Ulrich Baum. *Fast Fourier Transforms*. BI-Wissenschaftsverlag, Mannheim, Germany, 1993.

> ANNOTATION: This text provides an excellent introduction to the theory of generalized FFTs as of 1993. It discusses the necessary background in group algebras and their representations and in linear complexity theory and then provides FFTs for abelian groups, solvable groups, supersolvable groups, and the symmetric groups. The symmetric group algorithms it provides are those found in Clausen and Baum's 1993 article [5].

[5] Michael Clausen and Ulrich Baum. Fast fourier transforms for symmetric groups: Theory and implementation. *Mathematics of Computation*, 61(204):833–847, October 1993.

ANNOTATION: Clausen and Baum present an implementable algorithm for an FFT and an inverse FFT on the symmetric group S_n that requires $\frac{1}{2}(n^3 + n^2)n!$ operations. This implementation realizes the upper bound on linear complexity that Clausen specified in his 1989 article [2]. The algorithm relies on a left-coset factorization of S_n.

[6] Persi Diaconis. A generalization of spectral analysis with application to ranked data. *The Annals of Statistics*, 17(3):949–979, 1989.

ANNOTATION: Diaconis presents a means of analyzing both fully and partially ranked data using the representation theory of the symmetric group, and in particular to identify higher-order effects in ranking preferences. He applies these techniques to voting data from the APA presidential elections as well as from other sources. Finally, he investigates the statistical significance of the analysis he presents. Also includes an appendix pertaining to the decomposition of symmetry spaces into isotypics.

[7] David Dummit and Richard Foote. *Abstract Algebra*. John Wiley and Sons, New York, 1999.

ANNOTATION: Standard advanced undergraduate or first-year graduate student text on algebra, covering the basic theory of groups, rings, and fields along with additional advanced topics. Most useful for the material on module theory (Chs. 10–12) and representation theory (Chs. 18–19).

[8] Kenneth I. Gross. On the evolution of noncommutative harmonic analysis. *Am. Math. Monthly*, 85:525–548, August 1978.

ANNOTATION: Gross provides an overview of the development of noncommutative harmonic analysis. He covers its beginnings in classical Fourier series, its generalizations to functions on finite groups, the Peter-Weyl Theorem and harmonic analysis on compact Lie groups, the special case of spherical harmonics, classical harmonic analysis on noncompact domains, and applications to quantum mechanics.

[9] Gordon James and Adalbert Kerber. *The Representation Theory of the Symmetric Group*. Addison-Wesley, Reading, MA, 1981.

ANNOTATION: This text discusses the classic representations of S_n as determined by Alfred Young. Of particular interest to this project is Ch. 3, which derives the ordinary irreducible matrix representations of S_n from the decomposition of its group algebra through the seminormal basis of the algebra. Also potentially applicable is the discussion of the representation of wreath products of S_n, some of which correspond to the Weyl groups.

[10] David K. Maslen. The efficient computation of fourier transforms on the symmetric group. *Mathematics of Computation*, 67(223):1121–1147, July 1998.

ANNOTATION: Maslen presents an algorithm for an $O(n^2|S_n|)$ FFT algorithm for the symmetric groups S_n, which improves upon the results Clausen and Baum give in their 1993 article [5]. He accomplishes this by summing over the Young tableaux associated with the representations of S_n; presumably it is these sums over these combinatorial objects that render the algorithms so difficult to put into practice.

[11] David K. Maslen, Michael E. Orrison, and Daniel N. Rockmore. Computing isotypic projections with the Lanczos iteration. *SIAM J. Matrix Anal. Appl.*, 25(3):784–803, 2004.

ANNOTATION: Presents material on isotypic projections and their applications to decimation-in-frequency FFTs similar to that found in [16].

[12] David K. Maslen and Daniel N. Rockmore. Generalized FFTs - a survey of some recent results. *DIMACS Series in Discrete Mathematics and Computer Science*, 28:183–237, 1997.

ANNOTATION: Surveys results pertaining to the generalization of FFTs in a group-theoretic context. Discusses FFTs on abelian groups, on non-abelian finite groups, and on band-limited functions on compact Lie groups. Also covers results on fast discrete polynomial transforms. Closes with a set of open questions in the field.

[13] G. E. Murphy. A new construction of young's seminormal representation of the symmetric groups. *Journal of Algebra*, 69:287–297, 1981.

ANNOTATION: Murphy introduces a simple set of elements of KS_n that he uses to construct Young's seminormal representation of the symmetric group. These elements arise as differences of class sums of transpositions and hence are simultaneously diagonalizable. Murphy also uses these elements to reformulate proof of other relations pertaining to the representations of the symmetric group.

[14] G. E. Murphy. The idempotents of the symmetric group and nakayama's conjecture. *Journal of Algebra*, 81:58–265, 1983.

ANNOTATION: Murphy uses the elements L_u from his 1981 paper [13] to construct the primitive idempotents of KS_n for a field K of arbitrary characteristic and to demonstrate that the ring of symmetric function in L_u is equal to the center of KS_n. He further uses this framework to give a simple proof of Nakayama's Conjecture.

[15] Andrei Okounkov and Anatoly Vershik. A new approach to represention theory of symmetric groups. *Selecta Mathematica*, 2(4):581–605, 1996.

ANNOTATION: Okounkov and Vershik use Gelfand-Zetlin bases and Jucys-Murphy elements to construct the Young tableaux for the symmetric group more naturally. They generalize these techniques in the construction of similar characterizations for other Coxeter groups.

[16] Michael E. Orrison. *An eigenspace approach to decomposing representations of finite groups*. PhD thesis, Dartmouth College, 2001.

ANNOTATION: This document provides many of the basic theoretical techniques for determining decmation-in-frequency generalized FFTs, including the concept of projecting into isotypic subspaces with primitive idempotents. Discusses the computation of such projections through the Lanczos iteration. Gives several useful example involving the symmetric group, the hyperoctahedral group, and the finite general linear and symplectic groups.

[17] Arun Ram. Seminormal representations of Weyl groups and Iwahori-Hecke algebras. *Proc. London Math. Soc.*, 73:99–133, 1997.

ANNOTATION: Ram presents a method of determining representations for Weyl groups and Iwahori-Hecke algebras through a generalization of Young's seminormal representations for S_n. He accomplishes this by constructing Jucys-Murphy elements for these groups and algebras. Such representations are key for determining FFTs on general Weyl groups.

[18] Daniel N. Rockmore. Recent progress and applications in group FFTs. NATO Advanced Study Institute on Computational Noncommutative Algebra and Applications, July 2003.

ANNOTATION: Provides an accessible introduction to the state of generalized FFTs circa 2003. Includes good descriptions of both the Cooley-Tukey decimation-in-time FFT and the Gentleman-Sande decimation-in-frequency FFT as well as their generalization to nonabelian finite, compact, and noncompact groups.

[19] Bruce E. Sagan. *The Symmetric Group: Representations, Combinatorial Algorithms, and Symmetric Functions*. Wadsworth & Brooks/Cole, Pacific Grove, CA, 1991.

ANNOTATION: Provides an accessible survey of group representation theory, the representations of the symmetric group, combinatorial algorithms associated with these representations, and results and techniques involving symmetric functions. Of particular interest here is the representation theory of S_n, which Sagan approaches through an analysis of Specht modules.

[20] Jean-Pierre Serre. *Linear Representations of Finite Groups*. Number 42 in Graduate Texts in Mathematics. Springer, New York, 1977.

ANNOTATION: This is a classic text on representation theory and will be useful primarily as a reference. Many articles on the theoretical framework of generalized FFTs refer to this text for important theorems in representation theory.