Decimation-in-Frequency FFTs for the Symmetric Group: Problem Statement

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The discrete Fourier transform provides a way to convert samples of a periodic function into frequency information about that function, and consequently underlies much of modern signal processing theory. In recent years, significant attention has been paid to group-theoretic generalizations of the discrete Fourier transform and to their efficient implementation. In essence, these generalized discrete Fourier transforms correspond to a change of basis for the group algebra $\mathbb{C}G$ from the standard group-element basis to one in which the basis vectors represent frequency information. Thus, the DFT can be realized as the $|G| \times |G|$ matrix that performs the desired change of basis. Efficient algorithms for computing the DFT (Fast Fourier Transforms or FFTs) can then be related to sparse factorizations of this DFT matrix.

Much of the current research in generalized fast Fourier transforms for the symmetric group S_n has focused on separation of variables (decimation-in-time) algorithms. We intend to investigate projection-based (decimation-in-frquency) algorithms for the symmetric group, which may simplify both the theoretical framework for such FFTs and how such FFTs are implemented. One particular theoretical goal of this research is to use the representation thory of S_n and the projection-based framework it affords to construct decimation-in-frequency FFTs for S_n in terms of sparse factorizations of the DFT matrix. Another is to determine an upper bound on the linear complexity of these decimation-in-frequency FFTs, which we seek to do recursively by relating FFTs for large symmetric groups to those for smaller ones.

Finally, we seek to develop efficient implementations of these algorithms, which has not yet been accomplished for the most efficient decimation-in-time algorithms. In doing so, we plan to use a number of different mathematical software packages, including Mathematica, GAP, and Matlab. Mathematica presents a general framework for symbolic computation with a consistent programmatic interface. The GAP program (Groups, Algorithms, and Programming) affords an efficient means of working with linear and permutation representations of groups, and so is especially well suited to the representation theory of the symmetric group. Finally, Matlab is designed specifically to handle sparse matrix calculations efficiently; such calculations will certainly arise in both the computation and application of these sparse DFT matrix factorizations.