

# Research Proposal: Pebble Sets and Triangulations

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## 1 Introduction

After attending the colloquium by Randall Maddox entitled “Placing Points in Polygons”, Professor Su and I became interested in how pebble sets could be used to improve lower bounds of minimal triangulations of polytopes.

## 2 Proposed Research

In his lecture on October 20, 2004, Randall Maddox proved that any chamber in the pebble set of a polygon must have exactly three sides. I plan to examine pebble sets in polytopes to attempt to generalize these requirements for higher dimensions. In particular, I will consider the hypothesis that any chamber in a pebble set must be a simplotope. I plan to first classify pebble sets in three-dimensional objects, beginning with the cube.

In 2002, Su, de Loera, and Peterson [3] proved that any polytope with  $n$  vertices and  $d$  dimensions has a pebble set of size  $n - d$ , which means  $n - d$  is a lower bound on the minimal triangulation. I hope to find pebble sets that are larger than  $n - d$  for certain classes of polytopes by using the polytopal generalization of Sperner’s lemma, thus improving the lower bound. I plan to begin by studying simplotopes, the minimal triangulation of which was studied in 2004 by Seacrest and Su [2]. I will also consider the minimal triangulation of cubes in  $d$  dimensions. In 2003, Bliss and Su [1] studied the minimal triangulations for the 3-cube and the 4-cube, but the 5-cube is so complex that new methods are needed to feasibly improve upon the lower bound for its minimal triangulation, which is currently 60.

## References

- [1] Bliss, Adam & Su, Francis Edward, *Lower bounds for simplicial covers and triangulations of cubes*, To appear, Discrete and Computational Geometry.

- [2] Seacrest, Tyler & Su, Francis Edward, *Minimal triangulations of simplices*, In progress.
- [3] Su, Francis Edward, de Loera, Jesus, & Peterson, Elisha, *A polytopal generalization of Sperner's lemma*, J. Combinatorial Theory, Series A, 100 (2002) pp. 1-26.