

Research Proposal: An Exploration of Riemann's Zeta-function and it's Application to the Theory of Prime Distribution

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1 Introduction

I first read about the Riemann Hypothesis over 4 years ago. Since then I have been fascinated by the conjecture that all the nontrivial zeros of the Riemann zeta-function have real part $1/2$. This simply stated hypothesis has confounded mathematicians for nearly 150 years and stands as the foremost unproved conjecture in mathematics. I have decided to use my thesis as a means of becoming familiar with the Riemann zeta-function and understanding its relationship to the distribution of prime numbers. The literature on the Riemann zeta-function is immense, so to narrow my study I will devote the first month of effort to reviewing basic literature, particularly aspects bearing on prime numbers and the distribution of primes in arithmetic progressions. At that time, being better informed about the field, I will negotiate a specific work plan for the study with my thesis advisor. This procedure for confining work to within an achievable scope is practiced by the Mathematics Clinic, and it seems a good approach to making an informed decision concerning objectives for a thesis.

2 Proposed Research

Given the fact that the Riemann zeta-function has been a major topic of research in analysis for well over a century, it is most likely that my thesis will be primarily expository. At this point, I can see that candidates for my exposition could include a (comparative) study of proofs of the prime number theorem, the theta function used in some of these proofs, and Dirichlet's proof that given relatively prime integers a and b , the sequence $a, a + b, a + 2b, a + 3b, \dots$ contains an infinite number of primes. A recently proved result that there exist arithmetic progressions of primes of infinite length will also be considered.

My goal is to familiarize myself with some particular problem for which the Riemann zeta-function is applicable and present it so that anyone with a moderate mathematical background can understand: What the Riemann zeta-function is, Riemann's hypothesis about his zeta-function, and the applicability of the zeta-function to prime number distribution. In addition, I would like to see whether the Riemann zeta-function sheds new light on cryptographic security, since primes and the factorization of primes play roles in that area.

I expect that this thesis will provide me with a well-motivated introduction to complex analysis and analytic number theory. And, of course, I hope that I will find something original to contribute.

3 Prior Research

There is a huge amount of prior research on the Riemann zeta-function, since the Riemann Hypothesis has been under the mathematical microscope for over a century.

For background, I have already taken courses in analysis, discrete mathematics, cryptography, and I am currently studying Davenport's [3] which should provide a foundation in number theory. Concerning Riemann's zeta-function and infinite series, I have access to books by Knopp [4], Edwards [5], and Patterson [6]. I have already begun browsing through these books, and they will be among the resources for my research.

References

- [1] Richard K. Guy, *Unsolved Problems in Number Theory*, Springer-Verlag New York, 2004, pp. 25-27.
- [2] Terence Tao and Ben Green, "The Primes Contain Arbitrarily Long Arithmetic Progressions", Submitted to *Annals of Mathematics*, Nov 2004.
- [3] H. Davenport, *The Higher Arithmetic*, Cambridge University Press, Seventh Edition, 1999.
- [4] Konrad Knopp, *Theory and Application of Infinite Series*, Dover Publications New York, 1990.

- [5] H.M. Edwards, *Riemann's Zeta Function*, Dover Publications New York, 2001.
- [6] S. J. Patterson, *An Introduction to the Theory of the Riemann Zeta-Function*, Cambridge University Press, 1988.