

Research Proposal: Applications of the q -Binomial Coefficients to Counting Problems

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1 Introduction

The binomial coefficients are one of the central tools used to solve counting problems. The q -binomial coefficients are a generalization of the binomial coefficients with applications in diverse areas of mathematics.

A q -binomial coefficient is given by

$$\begin{bmatrix} n \\ m \end{bmatrix}_q \equiv \frac{(q)_n}{(q)_m (q)_{n-m}} = \prod_{i=0}^{m-1} \frac{1 - q^{n-i}}{1 - q^{i+1}},$$

where

$$(q)_k \equiv \prod_{m=1}^{\infty} \frac{1 - q^m}{1 - q^{k+m}}$$

2 Proposed Research

I will start my research by familiarizing myself with the existing literature on the subject of q -binomial coefficients. I'd like to focus my inquiry on applying the q -binomial coefficients to new problems. Throughout my initial study, I'll be looking for possible parallels and similarities with other fields of study which might suggest new areas in which the q -binomial coefficients can be applied. I'll also be keeping an eye open for existing combinatorial problems whose solution methods might be simplified with the q -binomial coefficients.

3 Prior Research

I don't have any previous experience with this particular topic. However, in both my Combinatorics and Probability classes, I've become familiar with using the binomial coefficients to solve a variety of counting problems. I've taken courses in cryptography and advanced abstract algebra, and I plan to take Number Theory in the fall. All of these courses should give me ideas for areas in which to apply the q -binomial coefficients.

References

[1] Eric W. Weisstein. "q-Binomial Coefficient." From *MathWorld*--A Wolfram Web Resource. <http://mathworld.wolfram.com/q-BinomialCoefficient.html>

[2] Andrews, G. E. *q-Series: Their Development and Application in Analysis, Number Theory, Combinatorics, Physics, and Computer Algebra*. Providence, RI: Amer. Math. Soc., p. 10, 1986.