Harvey Mudd College
Mathematics

Senior Thesis
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Research Proposal: Modeling Wave Propagation in Viscoelastic Fluids

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1 Introduction

I have have always enjoyed programming and was inspired by my scientific computing and dynamical systems classes last semester to gain research experience in an area of mathematics that will require a heavy amount of computer programming complimented by a great deal of analysis. This past summer I explored one model of viscoelastic flows in one dimension with Professor Jacobsen and made some excellent progress. I plan on using senior thesis as opportunity to explore related but different facets of the same model. We were able to break up all possible traveling wave solutions into three major categories, however we have not yet proven what happens when these waves interact, or why all solutions seem to converge to some traveling wave.

2 Proposed Research

For classical Newtonian fluids we can use Burger's equation to model wave propagation which is given in its general from by $u_t + uu_x = cu_{xx}$ where $u(x,t)$ represents the velocity of the fluid flow in one dimension. This area of study has been highly researched in the past, however if we modify the equation slightly to $u_t + uu_x = \sigma_x$ for some function $\sigma(x,t)$ the solutions to this model change significantly. The function σ describes the stress of the fluid and hence changes the form of the waves that are generated in this fluid as well as their propagation speed. In particular I will study functions σ which satisfy the equation $\sigma_t + u\sigma_x = (\sigma + \alpha^2)u_x - \beta\sigma$. I will be experimenting with different numerical algorithms for solving this PDE quickly and accurately. While I have already created a numerical scheme for this problem over the summer, I'm looking to make tremendous improvements on its accuracy and runtime. This program will be used as an aid for the analysis. If I'm lucky, perhaps we'll discover something interesting and innovative about the numerics as well.

References

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[3] Renardy, Michael, *Mathematical Analysis of Viscoelastic Flows*, Society for Industrial and Applied Mathematics, 2000.

[4] Knobel, Roger, *An Introduction to the Mathematical Theory of Waves*, American Mathematical Society, 2000.