

Locality & Complexity in Path Search

Andrew Hunter

Senior Thesis

May 4, 2009

Purpose

Networks are everywhere

- ▶ Telephone systems
- ▶ Routers

Networks are complicated

- ▶ Routing algorithms?
- ▶ Link failure?
- ▶ Excess traffic?

Locality &
Complexity in Path
Search

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Path Search

Motivation

The model

Results on F_k

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A (realism?) problem

Our revised model

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Conclusions

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Our goal

Analyze a simplified model of communication.

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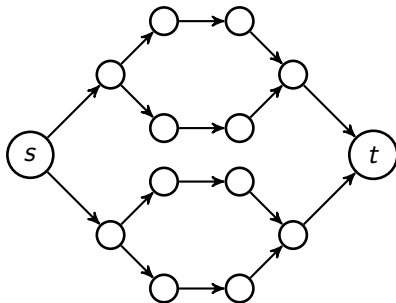
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Path search

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Channel graph G and free probability q



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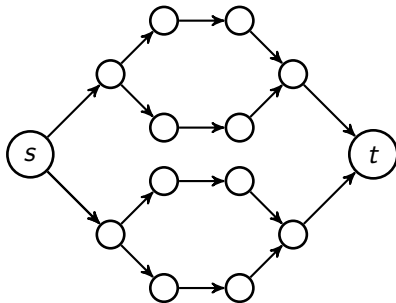
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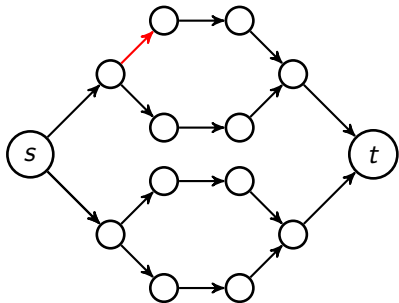
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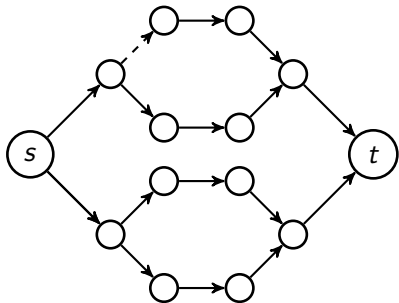
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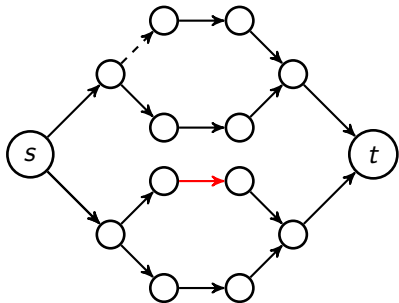
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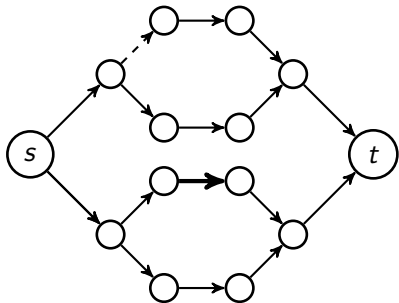
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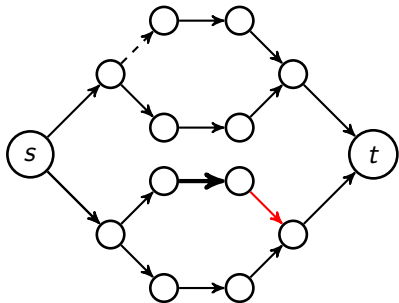
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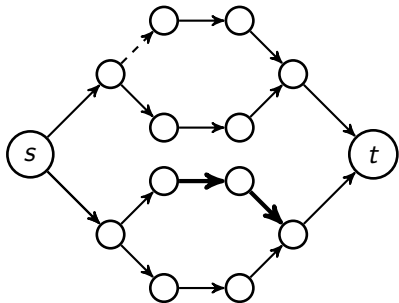
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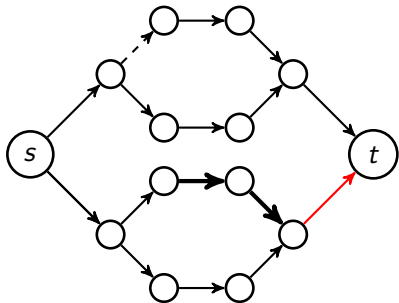
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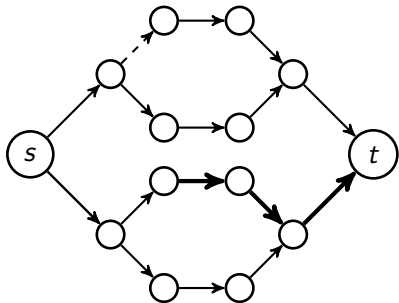
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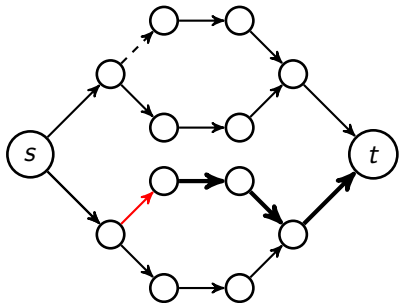
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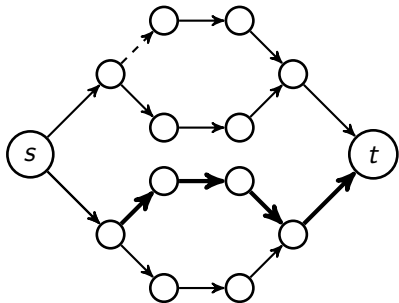
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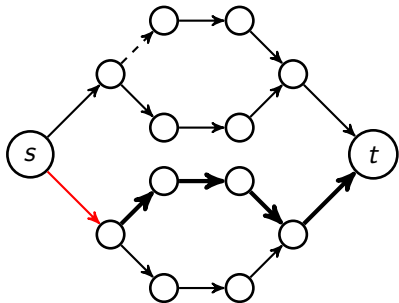
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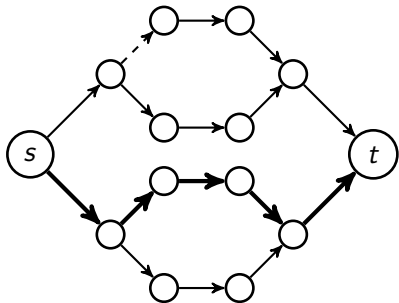
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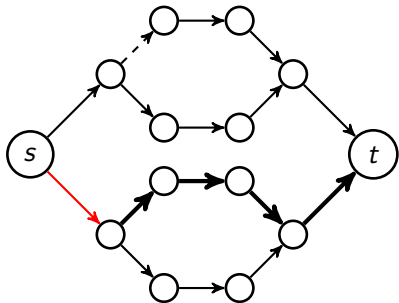
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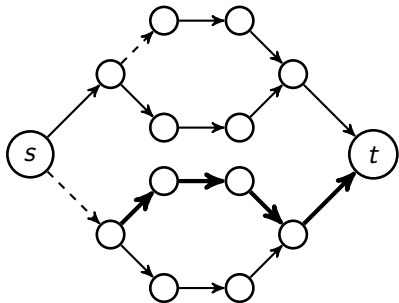
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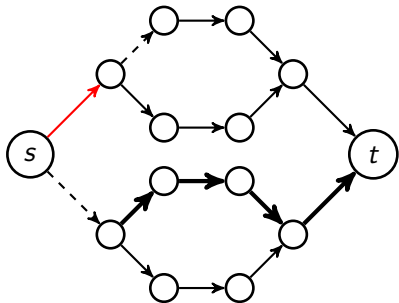
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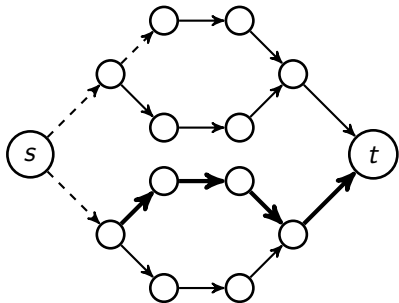
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Notation & definitions

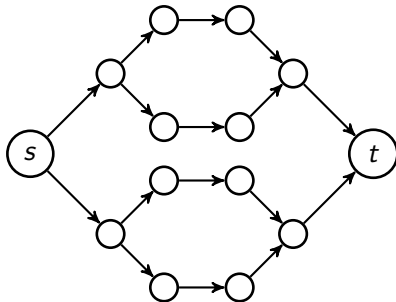
Channel graph	DAG with single source/sink
q	Free probability (edge)
p	blocked probability (edge)
$P_F(G, q)$	Free probability (graph)
$P_B(G, q)$	blocked probability (graph)
$E(G, q)$	Optimal search cost

Previous results

Theorem (Lin & Pippenger)

$$E(F_k, q) \in \Theta(k)$$

Proof idea.

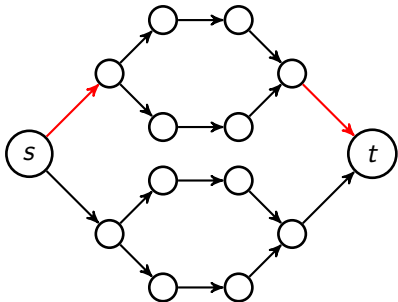


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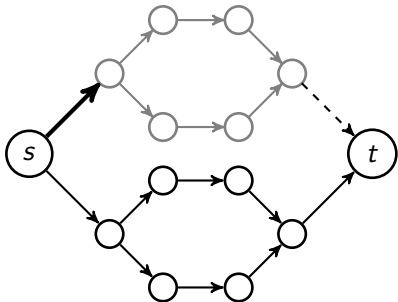


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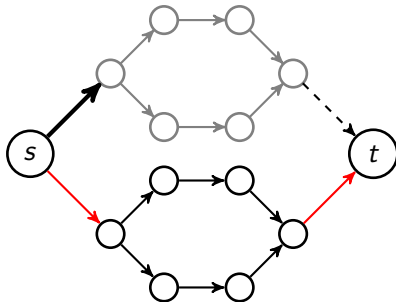


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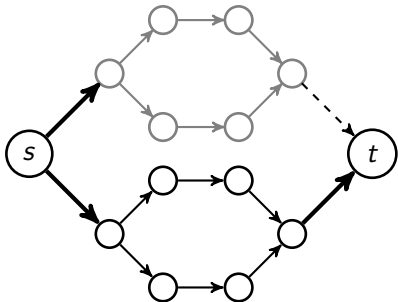


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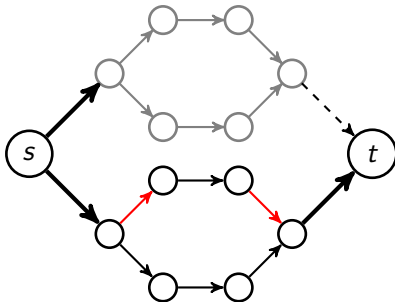


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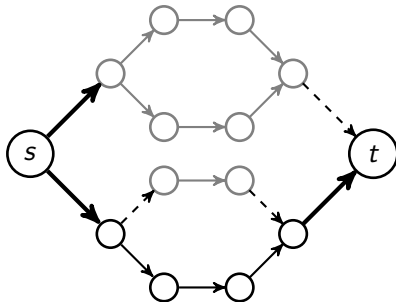


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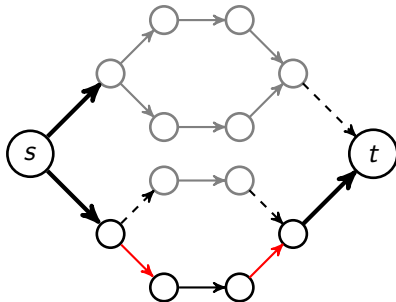


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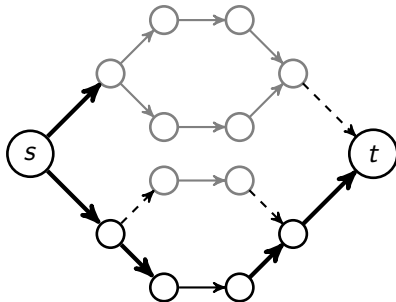


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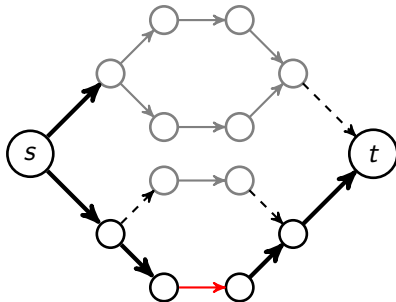


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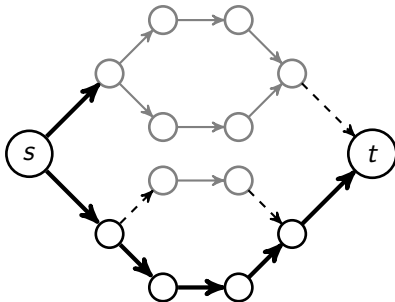


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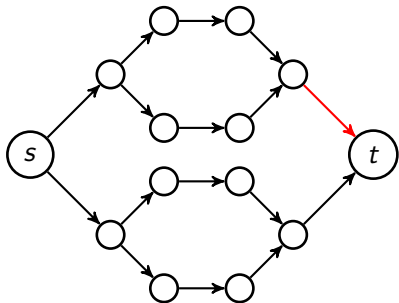
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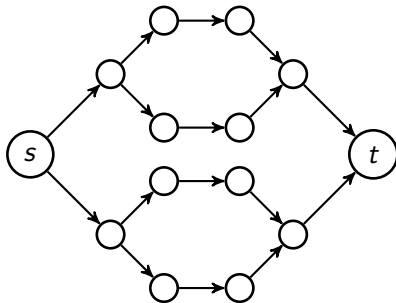


Does this match our network model?

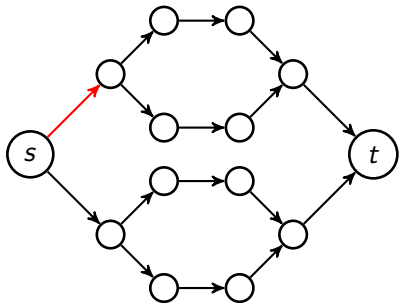


If I'm trying to talk to t , how do I possibly probe that edge?

What is feasible?



What is feasible?



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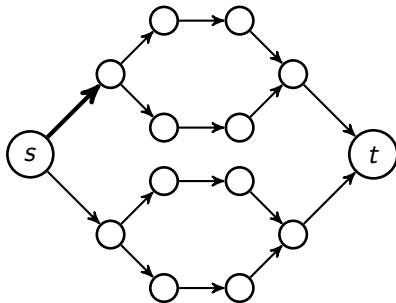
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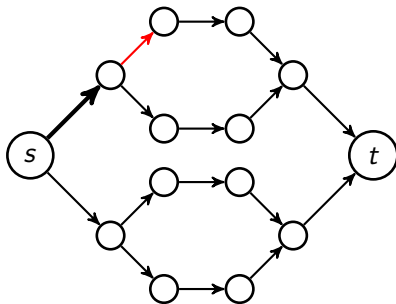
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What is feasible?



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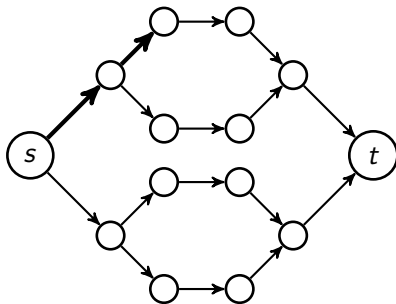
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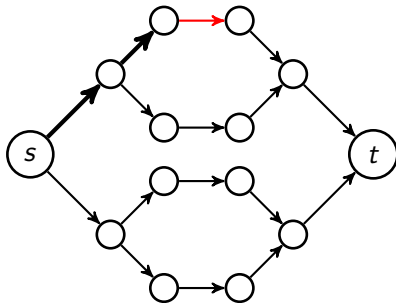
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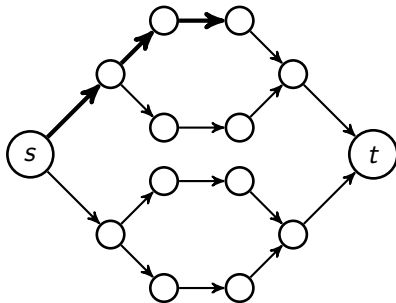
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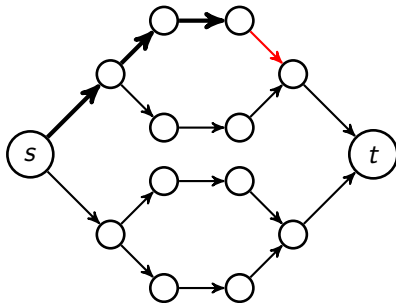
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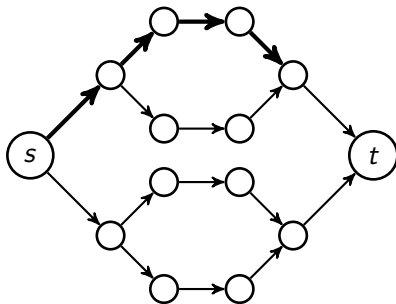
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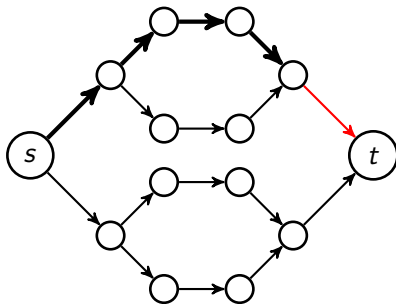
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What is feasible?



What is feasible?



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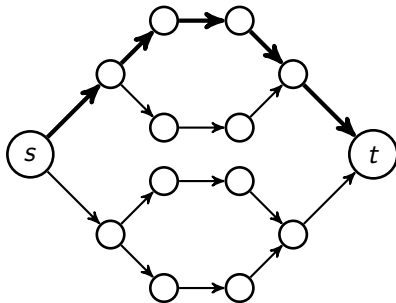
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What is feasible?



Local path search

Definition

In *local* path search, we may only probe edges we have a free path to.

Questions

How does this affect path search?

Feasibility

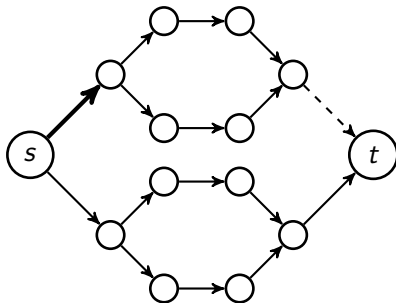
- ▶ All graphs still searchable
- ▶ $P_F(G, q)$

Tractability

- ▶ What about $E(G, q)$?
- ▶ Seems harder to perform searches

Why is local search harder?

It is much harder to rule out subgraphs.



How bad is DFS now?

Theorem

Searching F_k via DFS (locally) is

$$\begin{cases} O(k) & q < \frac{1}{2} \\ O((2q)^k) & \frac{1}{2} < q < \frac{1}{\sqrt{2}} \\ O\left(\frac{1}{q^k}\right) & \frac{1}{\sqrt{2}} < q \end{cases}$$

Can we do better?

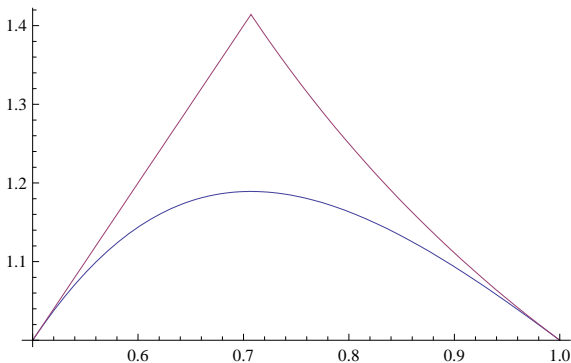
Lower bounds on F_k

Possibly, but not by much.

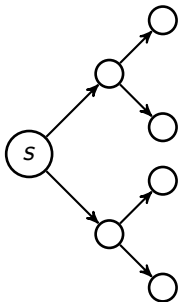
Theorem

For $q > \frac{1}{2}$,

$$E(F_k, q) \in \Omega \left(\left((2q)^{-\log q} \right)^k \right).$$



Lower bound for $\frac{1}{2} < q < \frac{1}{\sqrt{2}}$



Proof idea.

Probability of escaping the diverging part of T_k is very low, but a large part of it is accessible. \square

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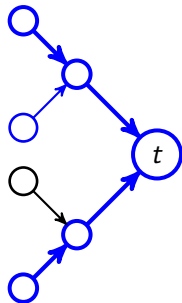
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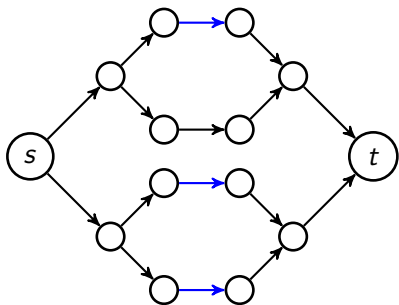
Lower bound for $\frac{1}{\sqrt{2}} < q$



Proof idea.

Complete binary trees are hard to search; the converging tree contains a (relatively deep) one. \square

Our main result



Proof idea.

Most of the leaves of the converging tree are available; any probe into that tree can only eliminate a few. □

What good is an unknown algorithm?

Our mental model

G has some optimal search algorithm A with cost $E(G, q)$.

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What good is an unknown algorithm?

Our mental model

G has some optimal search algorithm A with cost $E(G, q)$.

Lower bounds are hard!

How do we find that algorithm?

How do we know it is the best?

Complexity-theoretic path search results

Key observation

Answering many path-search related questions is (probably) very difficult.

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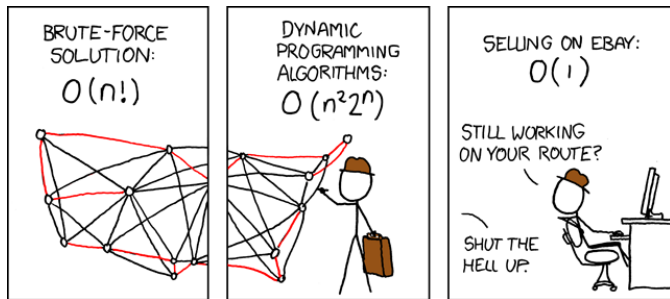
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Complexity-theoretic path search results

Key observation

Answering many path-search related questions is (probably) very difficult.



A crash course in complexity theory

Tractable problems

Intractable problems

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Search

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Path Search

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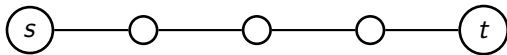
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Tractable problems

- ▶ Graph connectivity

Intractable problems

Or...



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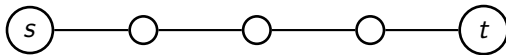
Tractable problems

- ▶ Graph connectivity

Intractable problems

- ▶ Graph partitioning

Or...



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Tractable problems

- ▶ Graph connectivity
- ▶ 2CNF satisfiability

Intractable problems

- ▶ Graph partitioning

Or...

$$(X \vee Y) \wedge (Z \vee \neg W) \wedge \dots \wedge (Y \vee \neg Z)$$

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Tractable problems

- ▶ Graph connectivity
- ▶ 2CNF satisfiability

Intractable problems

- ▶ Graph partitioning
- ▶ 3CNF satisfiability

Or...

$$(X \vee Y \vee Z) \wedge (Z \vee \neg W \vee A) \wedge \dots \wedge (Y \vee \neg Z \vee \neg D)$$

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Tractable problems

- ▶ Graph connectivity
- ▶ 2CNF satisfiability
- ▶ Linear programming

Intractable problems

- ▶ Graph partitioning
- ▶ 3CNF satisfiability

Or...

Maximize

$$c^T x$$

subject to

$$Ax \leq b$$

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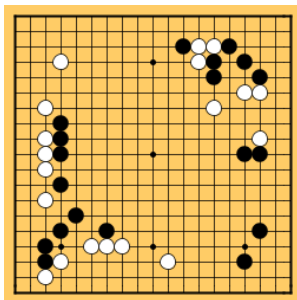
Tractable problems

- ▶ Graph connectivity
- ▶ 2CNF satisfiability
- ▶ Linear programming

Intractable problems

- ▶ Graph partitioning
- ▶ 3CNF satisfiability
- ▶ Optimal play in Go

Or...



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Tractable problems

- ▶ Graph connectivity
- ▶ 2CNF satisfiability
- ▶ Linear programming

Intractable problems

- ▶ Graph partitioning
- ▶ 3CNF satisfiability
- ▶ Optimal play in Go

Definition

A *complexity class* is a set of problems all solvable with some restricted set of resources.

The catch

Proving problems intractable is generally out of our reach.

Conjecture

$P \subset NP$.

Conjecture

$NP \subset PSPACE$.

Hardness

Definition

A problem X is hard for a complexity class C iff solving X easily lets us solve anything in C easily. Furthermore, if $X \in C$, we say it is C -complete.

The canonical example

Theorem (Cook-Levin)

3SAT is NP-complete.

Classes we'll need

Class	Resources	Meaning
P	Polynomial time	Tractable problems
NP	Polynomial time verifiers	Puzzles
$\#P$	Counting NP solutions	Reliability problems
$PSPACE$	Polynomial space	Games

Classes we'll need

Class	Examples
P	Linear programming, shortest path
NP	3SAT, TSP
$\#P$	Permanent, perfect matchings
$PSPACE$	Chess, Go, quantum computing simulations

Key building block

Theorem (Valiant)

Calculating $P_F(G, q)$ is $\#P$ -complete.

Proof Idea.

- ▶ If $q = \frac{1}{2}$, all subgraphs of G are equally likely
- ▶ Count connected subgraphs

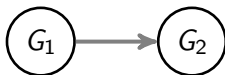


Cost is at least as hard

Theorem

Calculating $E(G, q)$ is #P-hard.

Proof Idea.



Succinct results

Succinct graphs

Instead of (V, E) , we have (C, n) (exponentially smaller)

Theorem

Computing $P_F(G, q)$ for succinct graphs is PSPACE-hard.

Proof Idea.

Follows Savitch's theorem. We can replace connected/disconnected graphs with probably linked/probably blocked graphs. □

Succinct results

Theorem

Computing $P_F(G, q)$ for succinct series-parallel graphs is PSPACE-complete.

Proof Idea.

Above graphs can be made series parallel, and probabilities are independent for disjoint parts of SP graphs. □

Open questions

Just how hard is search cost?

(We think) a gap exists between $\#P$ and $PSPACE$.

How much can we do on succinct graphs?

Only obvious algorithms (except series-parallel) are in $EXPSPACE$.

Locality's effect

Tasks are harder

$E(F_k, q)$ is exponential

Analyses are easier

We rely on locality in our
complexity reductions

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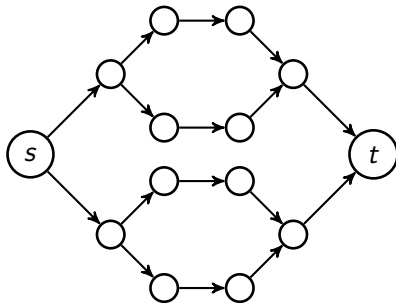
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Limits of locality



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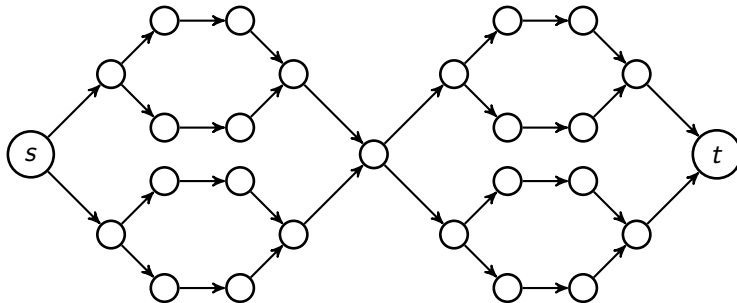
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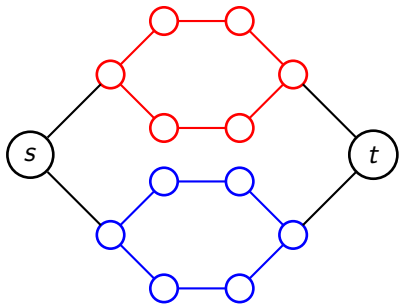
Open problems

Greediness

Is DFS optimal?

Parallel constructions

How can we combine costs from parallel subgraphs?



Complexity problems

Shrinking the
upper/lower gap

$\#P \subset PSPACE$ (probably)

Separating probabilities
and cost

Is it strictly harder to find
algorithms?

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Path search

Path search is simple, but rich—still models many of the facets of networks

Locality

Restricting our model has made it considerably *more* interesting.

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Questions?