Harvey Mudd College
Mathematics

Senior Thesis
Fall, 2008

Research Proposal: Generalized Criteria for No Continuous Solution of a Semilinear Wave Equation

Ben Preskill

Faculty Advisor: Professor Castro

1 Introduction

The semilinear wave equation

$$\Box(u) + g(u) = p(x,t) \quad x, t \in \mathbb{R}$$

has many applications throughout physics and mathematics. Investigating conditions under which the equation has continuous solutions is important in determining regularity and existence of meaningful results.

2 Proposed Research

I will investigate the equation

$$\begin{aligned} \Box(u) + g(u) &= p(x,t) = p(x,t+2\pi) = p(x+2\pi,t) \quad && x,t \in \mathbb{R} \\ u(x,t) &= u(x,t+2\pi) = u(x+2\pi,t) && x,t \in \mathbb{R} \end{aligned}$$

where $\Box$ denotes the D'Alambert operator $\partial_{tt} - \partial_{xx}$. Here

$$g(t) = \tau t + h(t)$$

where $h(t)$ is bounded and differentiable. In other words, g is asymptotically linear.

In particular, I will seek to prove that if g is asymptotically linear and nonmonotonic, then there exists $p(x,t)$ such that the semilinear wave equation has no continuous solution. In addition, I will seek to prove that if $h(t)$ has support in $[0, D]$ for some D such that

$$h(D/2) < -\tau D/2$$

then L^2 solutions do not exist. The result has already been shown for continuous solutions, and the validity is not yet known. It is possible that I will ultimately show L^2 solutions do exist.

3 Prior Research

My research will be adapted most directly from [2], in which it is established that, in the semilinear equation above, if $h(x)$ is differentiable and has support in $[0, D]$ such that

$$h(D/2) < -\tau D/2$$

then the semilinear wave equation has no continuous solutions.

It should be noted that in the case that g is monotonic, it has been shown in [3] that the semilinear wave equation will have solutions for p in a dense subset of L^2.

References

[1] H. Brezis, L. Nirenberg, *Characterizations of the ranges of some non-linear operators and applications to boundary value problems,* Annali della Scuola Norm. Sup di Pisa. (1978), pp. 225-236.

[2] J. Caicedo, A. Castro, *A Semilinear Wave Equation with Smooth Data and No Ressonance Having No Continuous Solution* [Not Yet Published].

[3] M. Willem, *Density of the range of potential operators*, Proc. Amer. Math. Soc. 83 (1981), no. 2, 341-344.