

Introduction

The human immunodeficiency virus (HIV) is one of the largest health problems the world is currently facing (Figure 1). The virus results in a compromised immune system, and eventually leads to immune system failure, known as auto immune deficiency syndrome (AIDS). Treatment is available, called anti-retroviral therapy (ART), but it is very expensive. Many resource-constrained countries are unable to meet the treatment needs of their infected populations and are forced to ration drugs. The World Health Organization (WHO) has outlined distribution strategies that it deems reasonable, one of which is the preferential treatment of disadvantaged populations (UNAIDS, 2005). We seek to test the WHO's claim that the preferential treatment of disadvantaged populations has a positive effect on the spread of HIV.

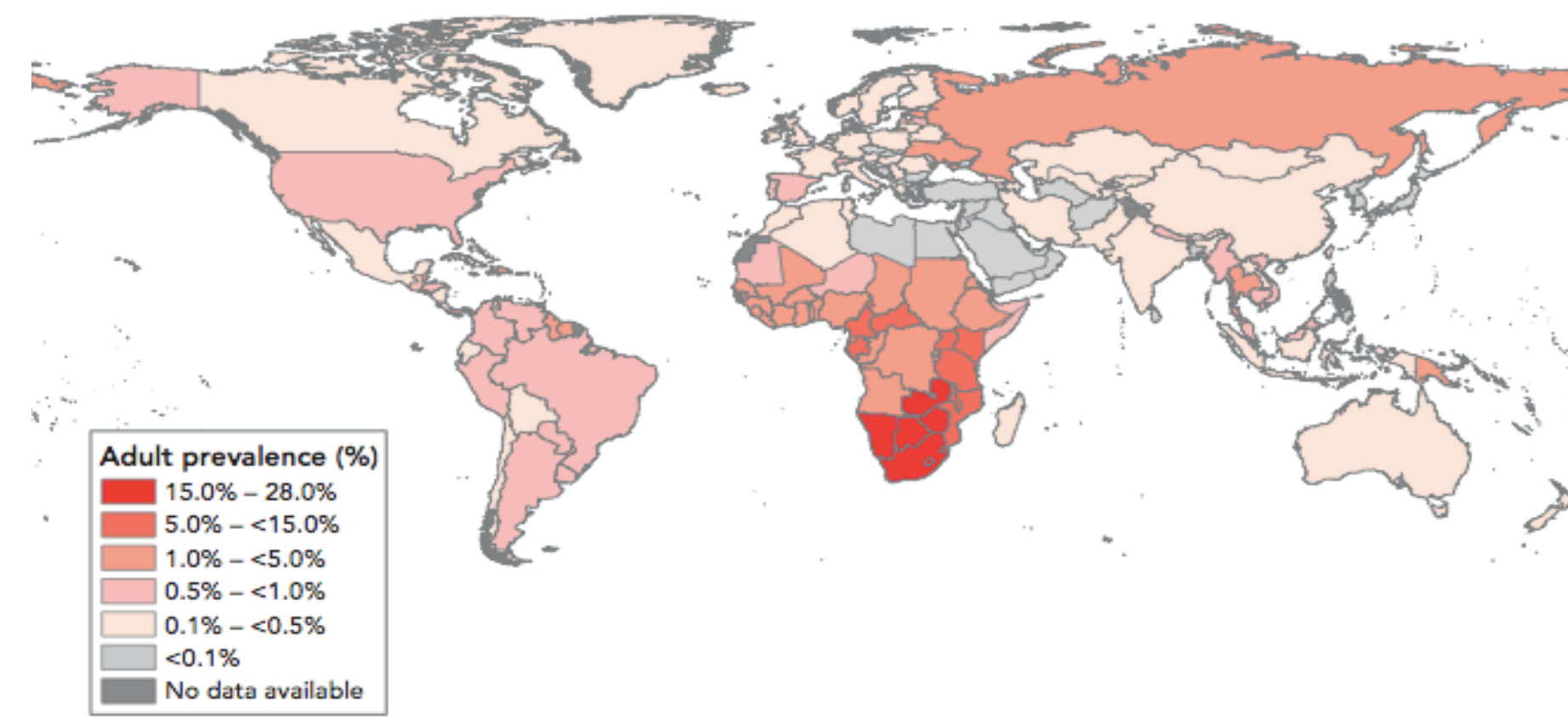


Figure 1: HIV positive people worldwide in 2007 (taken from UNAIDS (2008))

Model

In order to examine the effect of preferential treatment, we chose to create a two-sex multi-population compartmental model (Figure 2) that describes the flow of individuals through the stages of infection (susceptible (S), infected (I), treated (T), and AIDS (A)).

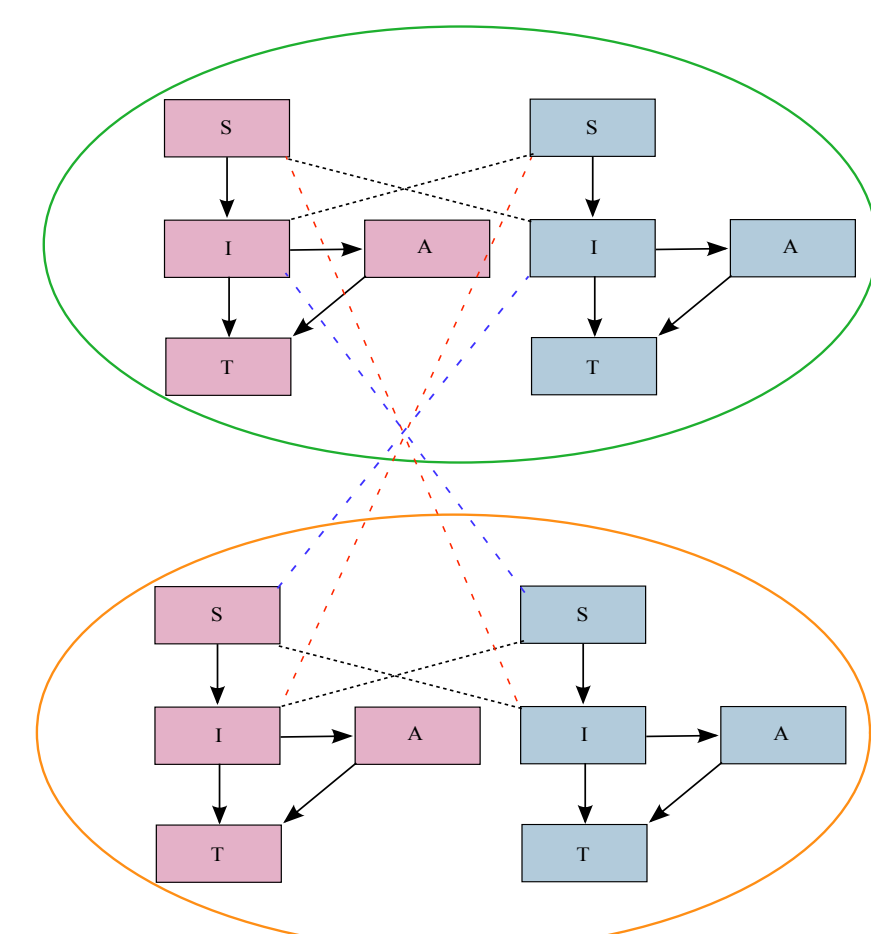


Figure 2: The total dynamics in the model, with dotted lines accounting for interactions leading to HIV infection.

The following equations, that describe the dynamics of women in the first population, exemplify the structure and form of the 16 differential equations that define our model:

$$\begin{aligned} \frac{dSW_1}{dt} &= bNW_1 - dSW_1 - \frac{\beta_{MW}SW_1(c_1IM_1 + c_2IM_2)}{NM_1 + NM_2} \\ \frac{dIW_1}{dt} &= \frac{\beta_{MW}SW_1(c_1IM_1 + c_2IM_2)}{NM_1 + NM_2} - (d + q)IW_1 - \tau_{W1}IW_1 - \alpha IW_1 \\ \frac{dAW_1}{dt} &= \alpha IW_1 - \tau_{W1}AW_1 - (l + d)AW_1 \\ \frac{dTW_1}{dt} &= \tau_{W1}IW_1 + \tau_{W1}AW_1 - dTW_1. \end{aligned}$$

Results

We ran a sensitivity analysis on the parameters, determined the basic reproductive number for our model, determined the stability of the equilibria, and performed simulations to determine the effect of preferential treatment.

Sensitivity Analysis

We have completed a preliminary Latin Hypercube Sensitivity analysis (LHS). The parameters β_{MW} and β_{WM} (transmission probabilities) ultimately have the largest effect on prevalence of HIV in 100 years.

Basic Reproductive Number

The basic reproductive number (R_0) of an epidemiological model is a parameter that helps determine whether a disease will become endemic ($R_0 > 1$) or die out ($R_0 < 1$) based on specific parameter combinations. The basic reproductive number was determined using the method described in van den Driessche and Watmough (2002). Our model simulations confirm the accuracy of R_0 determined through analysis (Figure 3), and the importance of the transmission probabilities (Figure 4).

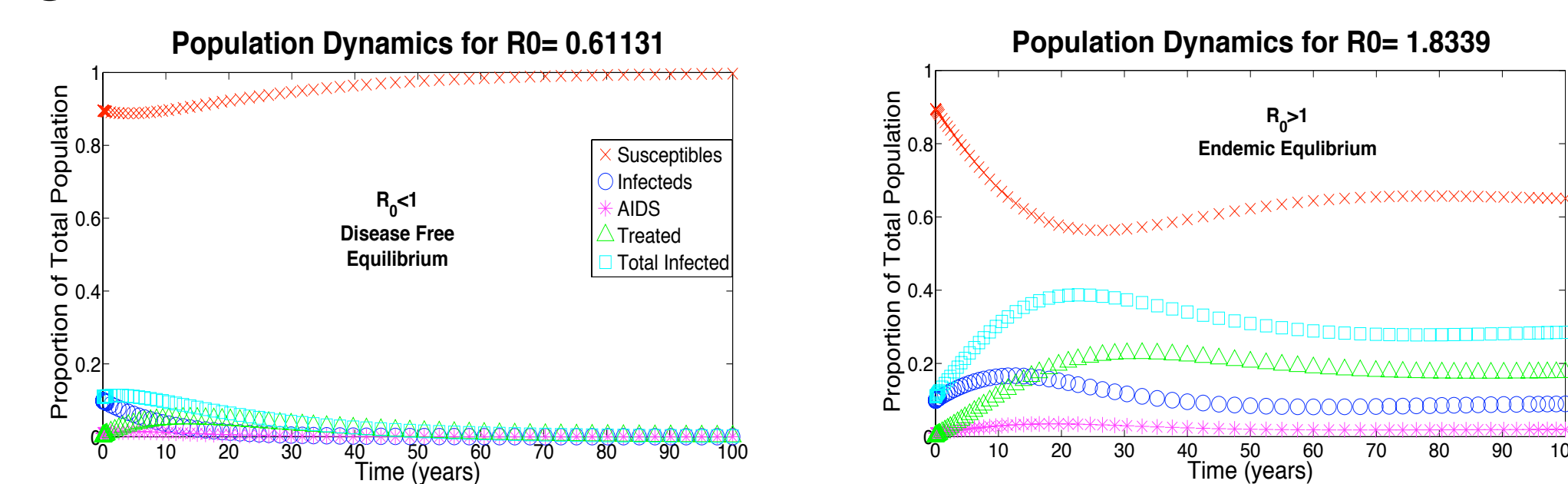


Figure 3: Simulation results exemplifying the accuracy of R_0 .

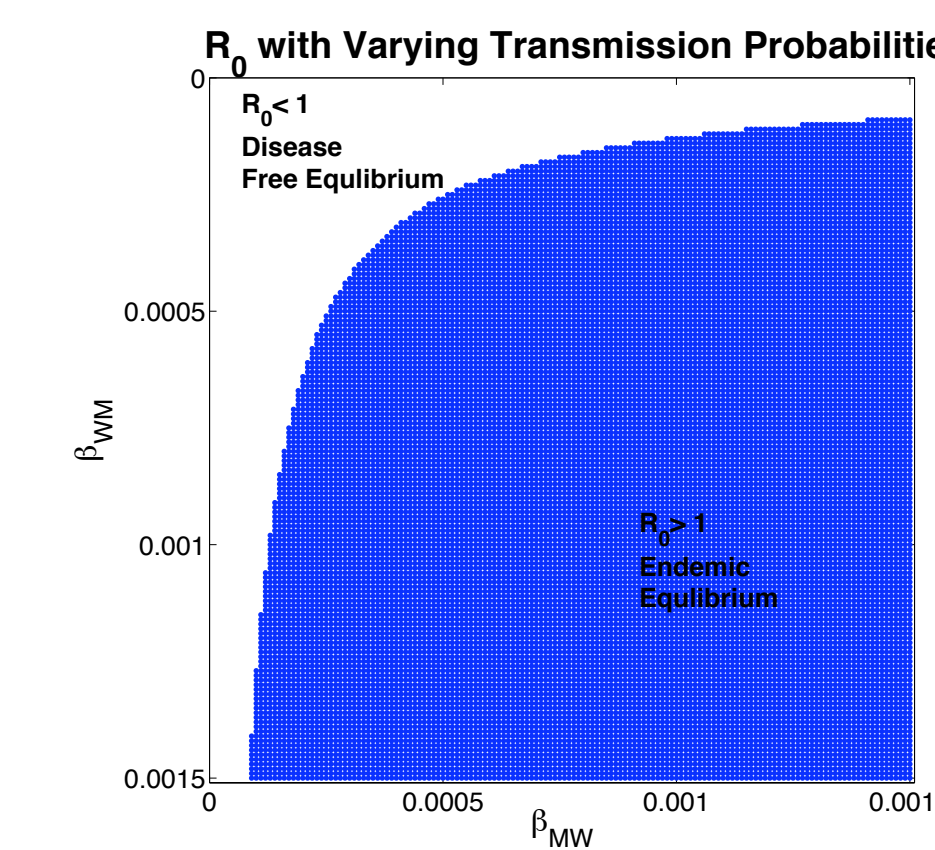


Figure 4: Stability of endemic equilibria over varying probability of transmission values.

Preferential Treatment: Simulations

We ran model simulations to determine whether the treatment of disadvantaged rural areas would have a positive effect on the future spread of HIV in both rural and urban populations.

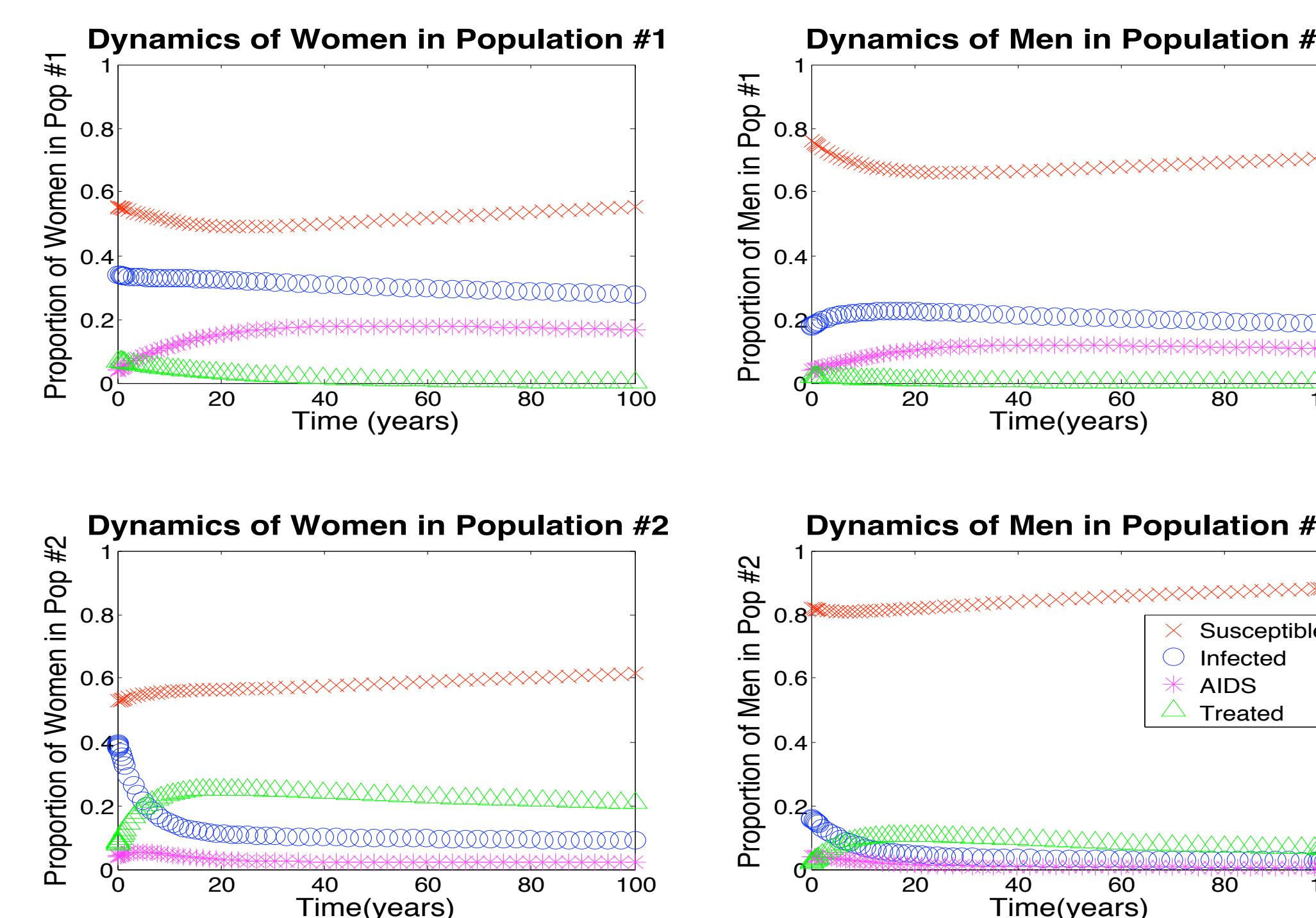


Figure 5: Preferential treatment to rural areas does not significantly reduce infection prevalence in all populations.

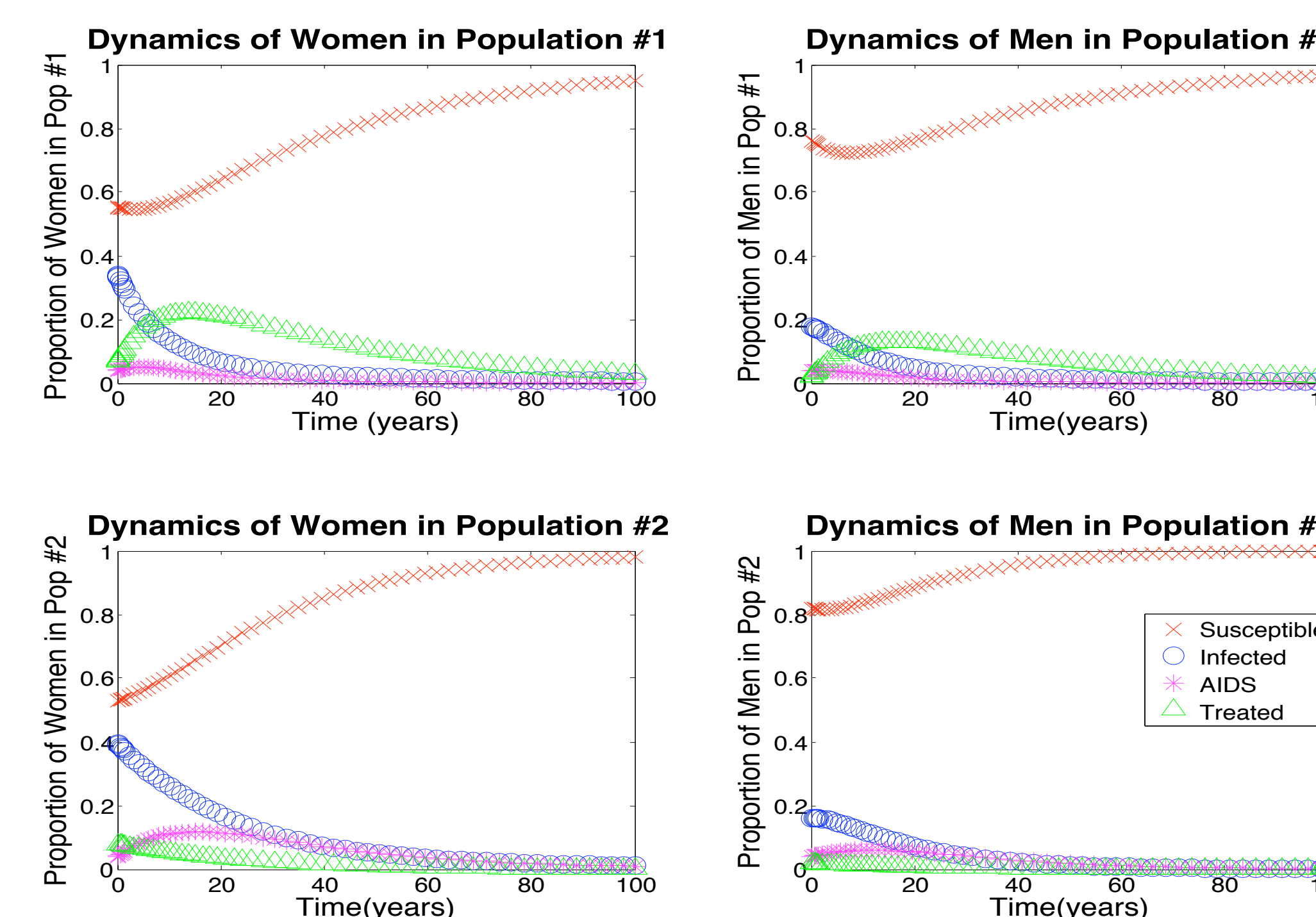


Figure 6: Concentrating treatment in urban areas has a large positive effect on all populations.

Conclusions

We conclude that concentrating treatment in disadvantaged rural areas does not significantly reduce the prevalence of HIV in all populations. In contrast, rationing treatment to urban areas positively affects all areas. We therefore cannot support, based on the current analysis, the WHO's recommendations for preferential treatment to disadvantaged populations in resource-constrained countries.

References

UNAIDS, WHO. 2005. Guidance on ethics and equitable access to HIV treatment and care. Tech. rep., WHO/UNAIDS.
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 van den Driessche, P., and J. Watmough. 2002. Reproduction numbers and sub-threshold endemic equilibria for compartmental models of disease transmission. *Mathematical Biosciences* 180(1-2):29–48.

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For Further Information

For more information on the project or further results, please feel free to contact the author at nabuelezam@hmc.edu. You may also visit the following website for full documentation: <http://www.math.hmc.edu/~nabuelezam/thesis/>.