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Research Proposal: Agreeability on Various Classes of Graphs

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1 Introduction

Given the real line and a collection of intervals on the real line, an interval graph can be constructed by viewing each interval as a node and connecting two nodes if the intervals they represent are non-empty. This can be viewed as a model of the situation where one has the typical conservative-liberal political spectrum with people who are comfortable with different intervals of the graph. In this scenario, the nodes then represent the people and edges represent the fact that two people's comfort zones overlap. By analyzing the graph it is possible to determine whether there are any positions that all people will be comfortable with, or agree on, or even that a majority of people will agree on. In [2] this scenario is analyzed and it is determined that in a society where in any group of m people there are some k of them that have a position they agree on then there is some position that at least $(k-1)/(m-1)$ of the society will agree on.

However, one could imagine situations where positions that people take do not simply lie on a line but are instead taken as a subset of a two (or higher) dimensional space. Alternately, positions could be subsets of a circle, or tree or possibly some more complicated graph. The higher dimensional cases is considered in [2] and [1] analyzes the case where the ambient space is a tree or cycle.

2 Proposed Research

I would like to look at what happens when the ambient space is a graph other than a tree or cycle, in particular I will most likely start by looking at the class of hypercube graphs. A logical starting place would be to look at the properties of graphs that result from applying the same procedure that results in interval graphs when the ambient space is just $\mathbb{R}^1$. Helly's Theorem states:

Given t convex sets in $\mathbb{R}^d$ where $d < t$, if every $d+1$ of them intersect at a common point, then they all intersect at a common point.

I will attempt to find similar results for taking subsets of the ambient graph.

3 Prior Research

When the ambient space is $\mathbb{R}^1$, the most useful technique in [2] seemed to be largely connecting graph theoretic ideas with applications of Helly's theorem. When considering trees and cycles, [1] brings topological and graph theoretic ideas together to prove Helly-like theorems which are then used to look at agreeability - a notion of how much a collection of the subsets of the ambient space overlap that considers the percentage of the subsets that have a non-empty intersection.

I took graph theory in the spring of my sophomore year and attended a geometric graph theory seminar last fall in Budapest. It will probably be useful to look into what work has been done on interval graphs and whether there has been any work done on generalizations of interval graphs. In addition, I am familiar with Helly's theorem and some ways it can be applied from the Conjecture and Proof course I took in Budapest. My background in topology is currently limited to what is covered in Analysis I and the little bits that have shown up in various classes, so I will be learning more topology.

References

[1] A. Niedermaier, D. Rizzolo, & F.E. Su, *A Combinatorial Approach to KKM Theorems on Metric Trees and Cycles*, Preprint, 2006.

[2] D. Berg, S. Norine, F.E. Su, R. Thomas,& P Wollan, *Set Intersections, Perfect Graphs, and Voting in Agreeable Societies*, Preprint, 2006.