Harvey Mudd College — Senior Thesis
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Research Proposal: Fast Fourier Transforms on Finite Groups

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1 Introduction

A discrete Fourier Transform (DFT) of a finite group G is an isomorphism

$$D : \mathbf{C}G \to \bigoplus_{i=1}^{k} \mathbf{C}^{d_i \times d_i},$$

that is, from the complex group algebra of G to a direct sum of algebras of $d_i \times d_i$ matrices, where k and the d_i are determined (up to order) by the structure of G. For example, when $G = \frac{\mathbf{Z}}{n\mathbf{X}}$, it turns out that $k = n$ and each $d_i = 1$; the map $\mathbf{C}(\frac{\mathbf{Z}}{n\mathbf{Z}} \to \bigoplus_{i=1}^{n} \mathbf{C}^{1\times 1} = \bigoplus_{i=1}^{n} \mathbf{C}$ is the standard DFT for periodic signals often encountered in engineering, where convolution on the left corresponds to componentwise multiplication on the right.

Evaluation of this isomorphism on an arbitrary element of $\mathbf{C}G$ by the obvious algorithm takes $O(|G|^2)$ time; a fast Fourier Transform (FFT) of G is an algorithm to evaluate the DFT in some smaller asymptotic time, for example $O(|G| \log |G|)$ or $O(|G| [\log |G|]^2)$.

2 Proposed Research

FFTs have been developed for several classes of groups (e.g., abelian groups, symmetric groups) using a decimation-in-time approach, which breaks down the work in time domain $\mathbf{C}G$.[1] I will instead attempt to develop and analyze FFTs using a decimation-in-frequency method, in which the work is broken down in the frequency domain $\bigoplus_{i=1}^{k} \mathbf{C}^{d_i \times d_i}$; I will consider these on some class of groups, such as wreath products by symmetric groups.

3 Prior Research

I have taken Algebra II (M172), in which I gained a general knowledge of module theory and DFTs.

References

[1] M. Clausen and U. Baum. *Fast Fourier Transforms.* BI-Wissenschaftsverlag, 1993.