

## **THE TWENTIETH INTERNATIONAL MATHEMATICAL OLYMPIAD**

**SAMUEL L. GREITZER**

The Twentieth International Mathematical Olympiad took place in Bucharest, Romania, on July 6 and 7, 1978. Thanks to the Office of Naval Research, our team had its three-week training session at the U.S. Naval Academy, Annapolis. The Army Research Office provided funds for our travel, and on June 30 we started our journey. Unfortunately, a long delay at Dulles Airport resulted in our arriving at Frankfurt too late to make our connecting flight to Bucharest, where we arrived one day late.

As has been usual, your delegate and secretary were separated from the team members and transported to the small town of Busteni. The selection of the six problems for the Olympiad began on July 3 and was completed on July 5. In spite of United States objections against having more than one problem from any one nation on the contest, the jury did select two problems from one country—the United States!

The Olympiad itself took place in Bucharest, at the Institutul Agronomic. There were impressive opening ceremonies on July 6 at which notables from Romania spoke, welcoming the contestants. Then the contest began. The problems will be found at the end of this article.

Grading of the problems actually began the same evening. Murray Klamkin and I completed our work by July 9, and even graded the two geometry problems for the Romanian students. On July 11, we were off to Bucharest, where we also stayed at the Institutul Agronomic. Our final meeting, with farewell ceremonies and a “solemn” dinner, took place on July 12 at the Institute. Our return trip on July 14 was without incident.

Our first surprise on arriving in Bucharest was to learn that the Soviet Union, Hungary, and the German Democratic Republic would not participate. Hungary did send a representative, who explained that the invitation to them had arrived too late. There was no word from either of the other two countries. We all had our own ideas as to why this situation had occurred. I myself managed to think of six reasons, most political in nature. At any rate, seventeen nations did take part. A list of the nations with their scores is appended. It should be noted that the United States was second, behind Romania, the host nation.

Austria	174	Germany	184	Romania	237
Bulgaria	182	Great Britain	201	Sweden	117
Cuba (4)	68	Mongolia	61	Turkey	66
Czechoslovakia	195	Netherlands	157	United States	225
France	179	Poland	156	Vietnam	200
Finland	118			Yugoslavia	171

Our second surprise was the participation, for the first time, of Turkey, and our third was the excellent showing of Vietnam, which had a complete team of eight students for the first time.

Our team consisted of the following students:

Andrew Bernoff	Upper Dublin H.S.	Fort Washington, Pa.
Daniel Bloch	Bellport H.S.	Brookhaven, N.Y.
Randall Dougherty	W. T. Woodson H.S.	Fairfax, Va.
Mark Kleiman	Stuyvesant H.S.	New York City, N.Y.
Victor Milenkovic	New Trier East H.S.	Winnetka, Ill.
David Montana	Lawrenceville School	Lawrenceville, N.J.
Ehud Reiter	T. S. Wootton H.S.	Rockville, Md.
Charles Walter	Centennial H.S.	Champaign, Ill.

The students performed nobly. First, Mark Kleiman had the only perfect score among all the contestants. Next, our team earned one “First Place” award, three “Second Place” awards, and three “Third Place” awards. The table below shows how our team performed, problem by problem. Researchers may draw some conclusions from comparison of the scores with the type of problem.

Our final surprise occurred at our farewell meeting on July 12, when we were informed that the 1979 Olympiad would be held in Great Britain.

Our own plans at present call for the Annual High School Mathematics Exam, which starts the Olympiad process, to take place on March 6, 1979, the U.S.A. Olympiad to take place on May 1, the training session to be at West Point in June, and the Twenty-first International Olympiad in early July. Meanwhile discussions are under way about the possibility of holding the International Olympiad in the United States in 1981.

Student	1	2	3	4	5	6	Total
Bernoff	3	4	4	5	6	0	22
Bloch	6	4	1	4	6	0	21
Dougherty	6	4	8	5	6	0	29
Kleiman	6	7	8	5	6	8	40
Milenkovic	5	7	6	5	6	1	30
Montana	6	4	4	5	6	1	26
Reiter	6	6	8	5	6	0	31
Walter	6	0	8	5	6	1	26
							225

**Twentieth International Mathematical Olympiad**

**Budapest, July 6–7, 1978**

**First Day**

- $m$  and  $n$  are natural numbers with  $n > m > 1$ . In their decimal representation, the last three digits of  $1978^m$  are equal, respectively, to the last three digits of  $1978^n$ . Find  $m$  and  $n$  such that  $m + n$  has its least value. (Cuba)
- $P$  is a given point inside a given sphere and  $A, B, C$ , are any three points on the sphere such that  $PA$ ,  $PB$ , and  $PC$  are mutually perpendicular. Let  $Q$  be the vertex diagonally opposite to  $P$  in the parallelepiped determined by  $PA$ ,  $PB$ , and  $PC$ . Find the locus of  $Q$ . (United States)
- The set of all positive integers is the union of two disjoint subsets  $\{f(1), f(2), \dots, f(n), \dots\}$ ,  $\{g(1), g(2), \dots, g(n), \dots\}$  where

$$f(1) < f(2) < \dots < f(n) < \dots,$$

$$g(1) < g(2) < \dots < g(n) < \dots,$$

and  $g(n) = f(f(n)) + 1$  for all  $n > 1$ . Determine  $f(240)$ . (Great Britain)

**Second Day**

- In triangle  $ABC$ ,  $AB = AC$ . A circle is tangent internally to the circumcircle of triangle  $ABC$  and also to sides  $AB, AC$  at  $P, Q$ , respectively. Prove that the midpoint of segment  $PQ$  is the center of the incircle of triangle  $ABC$ . (United States)
- Let  $\{a_k\}$  ( $k = 1, 2, 3, \dots, n, \dots$ ) be a sequence of distinct positive integers. Prove that, for all natural numbers  $n$ ,

$$\sum_{k=1}^n \frac{a_k}{k^2} > \sum_{k=1}^n \frac{1}{k}. \quad (\text{France})$$

- An international society has its members from six different countries. The list of members contains 1978 names, numbered  $1, 2, \dots, 1978$ . Prove that there is at least one member whose number is the sum of two members from his own country, or twice as large as the number of one member from his own country. (Netherlands)

DEPARTMENT OF MATHEMATICS, RUTGERS UNIVERSITY, NEW BRUNSWICK, NJ 08903.

**THE WILLIAM LOWELL PUTNAM MATHEMATICAL COMPETITION**

L. F. KLOSINSKI, G. L. ALEXANDERSON, A. P. HILLMAN

The following results of the thirty-ninth William Lowell Putnam Mathematical Competition, held on December 2, 1978, have been determined in accordance with the governing regulations. This annual contest is supported by the William Lowell Putnam Prize Fund for the Promotion of Scholarship, left by Mrs. Putnam in memory of her husband, and is held under the auspices of the Mathematical Association of America.