

Research Proposal: Biological Swarms and Chemoattractants

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1 Proposed Research

The modeling of swarming behavior is an emerging area of research in mathematical biology. For my thesis project, I intend to analyze the dynamics and stability of a particular class of swarm models in which the swarm members repel one another but are attracted to a point-source of chemoattractant at the origin. In one proposed model, the organisms in a swarm are confined to a one-dimensional domain, and their motion is governed by

$$\frac{dx_i}{dt} = F_c(x_i) + \sum_{j \neq i} F_s(x_i - x_j), \quad i = 1, \dots, N,$$

where $x_i(t)$ represents the position of the i th organism at time t . The functions F_c and F_s describe, respectively, the chemoattractive and social repulsive forces experienced by the i th organism. In this model, F_c is of the form

$$F_c(r) = -\gamma B \cdot \text{sgn}(r) \cdot |r|^{\gamma-1}$$

for some constants $\gamma > 0$ and $B > 0$, and F_s is of the form

$$F_s(r) = \beta \cdot \text{sgn}(r) \cdot |r|^{\beta-1} \cdot e^{-|r|^\beta}$$

for some constant $\beta > 0$.

This system approaches a steady state as $t \rightarrow \infty$, and the root mean square of the equilibrium positions can be used to measure the width of the swarm, with x_{rms}/N measuring the inter-organism spacing. The limit of the inter-organism spacing as $N \rightarrow \infty$ can be used to distinguish sets of parameters that yield qualitatively different behaviors.

In my thesis project, I plan to investigate the characteristics of such systems for large swarm sizes, eventually studying the model in higher dimensions and in the limit where the swarm approaches a continuum.

2 Prior Research

In an independent study with Professor Bernoff, I have worked on programming a Matlab simulation of the one-dimensional model described above. I have also begun reading the references listed below, which should help me understand this problem in a broader context. As for coursework, I am currently enrolled in Scientific Computing, and I plan to enroll in applied math courses such as Mathematical Biology and Dynamical Systems to learn new techniques for analyzing this problem.

References

- [1] A. Mogilner et al., *Mutual interactions, potentials, and individual distance in a social aggregation*, Journal of Mathematical Biology, 47 (2003) pp. 353-389.
- [2] D. Ruelle, *Statistical Mechanics: Rigorous Results*, W. A. Benjamin Publishers, 1969.