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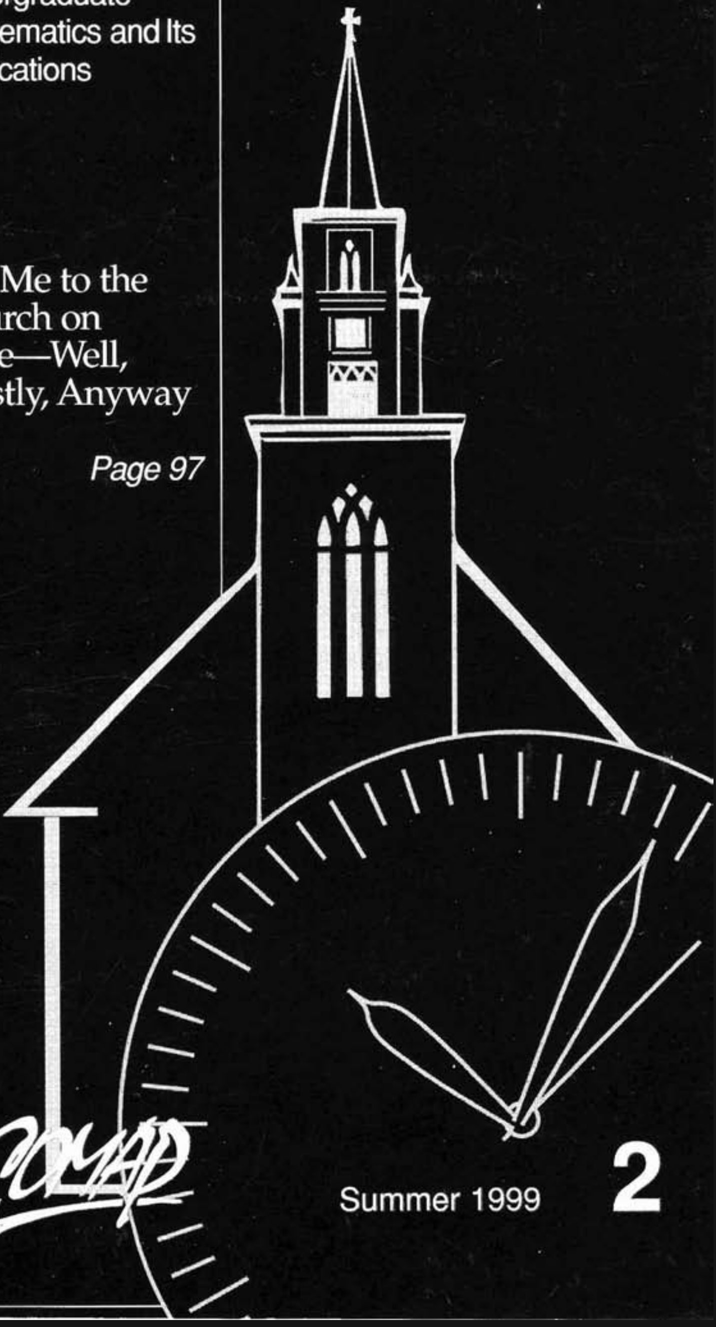

Get Me to the Church on Time—Well, Mostly, Anyway

Page 97

COMAP

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Why the Player Never Wins in the Long Run at LA Blackjack

Arthur T. Benjamin
Mike Lauzon
Chris Moore
Dept. of Mathematics
Harvey Mudd College
Claremont, CA 91711
benjamin@hmc.edu

Introduction

To get around an old California law that prohibits the game of "21," California card casinos introduced a variation of standard Blackjack, called *LA Blackjack,* in which the objective is to get as close as possible to *22* without going over.

The standard game of Blackjack, or "21", pits the player against the dealer ("the house"). Money lost by the player represents an equivalent gain by the house, and vice versa. In contrast, LA Blackjack pits the player against other players. Each player gets a turn to be the "banker" and collect money from the other players at the table, while the house collects a percentage of the total money bet. Mathematicians developed optimal plays for standard Blackjack many years ago, while the optimal plays for LA Blackjack are largely unexplored.

We analyze this game, develop a basic strategy for the player, explore the possibility of employing bet variance to raise the player's expected return, and conclude that a player cannot develop a profitable strategy.

The Rules of the Game

The game of LA Blackjack differs in many ways from standard Blackjack. First, there are three different actors in the game:

- The *dealer* is a casino employee who deals the cards and collects fees from each player.

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- The *banker* is a player who functions as "the house" in standard Blackjack. When a player wins money, the banker must cover the win out of his own bankroll. When a player loses money, the banker collects it.

- A *player* is any other person playing at the table.

The game is played from a six-deck shoe, which includes twelve Jokers that can be used as either 2 or 12 (just as an Ace can be used as either 1 or 11).

- The best hand is a *natural*, which consists of two Aces (denoted "A").

  The next-best hand is a Joker (denoted "J") plus any 10-value card (denoted "10", "T", or simply "Ten").

- The third-best hand is any three or more cards totaling 22.

  The remaining hands rank in getting as close to 22 as possible without going over. So, for example, 21 beats 20, 19 beats 18, and so on.

  A hand under 22 always beats a hand over 22; but should both player and banker exceed 22, then the lower of the two wins. So, for example, 23 beats 24 but 13 beats 23.

- Hands of the same rank are ties (called *pushes*), except for ties of 18, which are won by the banker (therein lies the banker's advantage).

- The rank of a hand can be either *soft* or *hard*, depending on whether it contains Aces and Jokers or not. For example, a hand containing an Ace and an Eight can total either 9 or 19. For our purposes, we refer to a hand that can have a value of n or $n + 10$ as a *soft* n hand.

The plays of the banker are forced. The banker must hit

- until a hard 18, then must stand;

- a soft 8 but must stand on soft 9 or higher.

The player has more discretion. The player

- may hit or stand on hard 13 through 17;

- may hit or stand on soft 3 through 7;

- must hit soft 8 (but may stand on lower soft totals);

- must stand on soft 9 or higher;

- must stand on hard 18 or higher.

Additionally, unlike standard Blackjack, in LA Blackjack there are no options to double down or to split pairs.

The LA Blackjack table has seven spots; one is for the banker, and the other six are ordered clockwise from the banker's left. A player may play more than one spot simultaneously.

Each player pays a fee to bet. The fee is $1 for a bet between $10 and $100; each multiple of $100 costs an additional $1, including a $1 fee on a partial multiple of $100. Thus, a $430 bet incurs a $5 fee but a $400 bet incurs only a $4 fee. The maximum bet allowed is $600.

The banker pays a flat fee of $2 to *put out* (cover the bets of players) any amount of money. The banker need not cover the bets of the entire table, but the bets must be covered in order of seating to the left of the banker. Any player not reached because the banker does not put out enough money is *out of action,* meaning the player can neither win nor lose. If the bank covers only a portion of the player's bet, then only that portion is *in action.* A push does not deplete the banker's total. So, for example, assume that the banker has a card total of 20 and puts out $300. Player 1 bets $75 and has 19. Player 2 bets $60 and has 20. Player 3 bets $40 and has 21. Player 4 bets $100 and has 22. Player 5 bets $100 and has 24. Player 6 bets $100 and has 21. As a result, Player 1 loses $75, player 2 pushes (which does not alter the amount of the banker's money that is in action), player 3 wins $40, player 4 wins $100, player 5 loses $85 (because only $85 of the bankers money remains in action at that point,) and player 6 wins $0 (this player is out of action because the amount put out by the banker is depleted).

All action starts to the left of the banker:

- All players receive two cards face up.
- All hits are dealt up.

The banker receives two cards, one up and one down.

- The banker acts after all other players act.

Methods

Determining the player's optimal strategy and its expected value took place in several steps. First, we devised a basic strategy for the player using an infinite-deck approximation of the game. While the game is played with six decks, it is far easier to work with an infinite deck, where the probability of receiving a card is always the same as it is in a full deck. That is, Jokers appear with probability 1/27, Aces through Nines each with probability 2/27, and Tens (including face cards) with probability 8/27. We calculated a table of the probability of a banker's final total given an original upcard. Then we used dynamic programming to determine if a player should hit or stand on any given total and banker upcard.

Once we had solved the infinite-deck game, we turned our attention to the finite deck and to card-counting strategies. Using Monte Carlo simulations, we first calculated the favorability of the game. Then we found the density of Tens needed to make the game favorable to a player who counts Tens. Finally, we simulated reduced decks (with specific cards removed) to estimate the change in deck favorability from removal of a single card. Using this information, we determined if any card-counting strategy could be profitable for the player.

In computing the dealer probabilities for infinite decks, we used two arrays, one for soft totals and the other for hard totals. We let

$$\begin{aligned} P[i,j] &= \text{the probability that the dealer ends with total } j \\ &\quad\ \text{given current hard total } i, \\ SP[i,j] &= \text{the probability that the dealer ends on total } j \\ &\quad\ \text{given current soft total } i \text{ (i.e., the total may be } i \text{ or } i+10) \end{aligned}$$

For base cases, we have for $i \geq 18$, $P[i,j] = 1$ if $i = j$ and 0 otherwise. For other values of i and j, we use the recursive formulae:

$$P[i,j] = \frac{1}{27}\left[8P[i+10,j] + 2SP[i+1,j] + SP[i+2,j] + \sum_{k=2}^{9}(2P[i+k,j])\right]$$

$$SP[i,j] = \begin{cases} P[i,j], & i \geq 13; \\ P[i+10,j], & 9 \leq i \leq 12; \\ \frac{1}{27}\Big(\sum_{k=2}^{9}(2SP[i+k,j]) & \\ \quad +8SP[i+10,j]+2SP[i+1,j] & \\ \quad +SP[i+2,j]\Big), & i \leq 8. \end{cases}$$

These equations were derived by conditioning on what the next card could be and taking a weighted average of those results. Each calculation of $P[i,j]$ depends only on the results where the total of the banker's cards are greater than the current banker's total. For example, to calculate $P[20,12]$, we need to calculate $SP[20,13]$—in case the banker receives an Ace—but knowledge of $P[20,11]$ is unnecessary; so we calculate higher banker holdings first.

The above calculation does not take into account AA or JT naturals. To correct for this omission, we calculate the probability of a natural for Ace, Joker, and Ten upcards:

$$\begin{aligned} P(\text{ AA} \mid \text{A upcard}) &= \frac{2}{27} = 0.07407 \\ P(\text{ JT} \mid \text{T upcard}) &= \frac{1}{27} = 0.03704. \\ P(\text{ JT} \mid \text{J upcard}) &= \frac{8}{27} = 0.29630. \end{aligned}$$

These equations lead to the following adjustments in our banker probability table:

$$P[22,10] \leftarrow P[22,10] - P(\text{JT} \mid \text{T upcard})$$
$$SP[22,2] \leftarrow SP[22,2] - P(\text{JT} \mid \text{J upcard})$$
$$SP[22,1] \leftarrow SP[22,1] - P(\text{AA} \mid \text{A upcard})$$

With a distribution of the dealer totals for any possible upcard, we can calculate a basic strategy for the player by using dynamic programming. For each combination of player totals and dealer upcards, we calculate the expectation as follows:

- If only one play is legal, such as standing on totals over 18, we assume that the player makes the legal play and calculate the expected value.
- If hitting and standing are both legal, we calculate the expected value for each action and record the greater value and corresponding action.

Calculating the expected value, E, for standing on a given total can be done by summing the probabilities of the banker having a total that beats the player, a total that ties the player, and a total that loses to the player, and then plugging the results into the following formula:[1]

$$E_{\text{stand}}[i,j] = 2P(\text{player wins}) + P(\text{player ties}) - 1.$$

This formula makes sense, since if you paid $1 to play, you would get back nothing if you lost, $1 if you tied, and $2 if you won. Now we must calculate the expected value of hitting. If the current card total cannot lead to a natural, we use the recursive formula

$$E_{\text{hit}}[i,j] = \frac{1}{27}\left(\sum_{k=2}^{9}(2E[i+k,j]) + 8E[i+10,j] + 2SE[i+1,j+1] + SE[i+2,j]\right);$$

and if the total is soft, we use

$$SE_{\text{hit}}[i,j] = \begin{cases} E[i,j], & i \geq 13; \\ E[i+10,j], & 9 \leq i \leq 12; \\ \frac{1}{27}\Big(\sum_{k=2}^{9}(2SE[i+k,j]) \\ \quad +8SE[i+10,j] \\ \quad +2SE[i+1,j] \\ \quad +SE[i+2,j]\Big), & i \leq 8 \end{cases}$$

[1] If we assume that we stand on a given total, there is no difference between a soft total and hard total. If we have a soft total, we define our final total to be the better of our two total choices.

We have to make exceptions to the formulae for totals that could lead to naturals: a 10 total that is a single card[2], or a soft 1 (an Ace), or a soft 2 (a Joker, since two Aces sum to a natural in our modified equations). A hard total of 10 may be a single card (*single 10*) or made up of several cards (*multi 10*). We can use the above formulae to calculate $E_{\text{hit}}[\text{multi } 10, j]$, using the multi 10 value in the formula for $SE_{\text{hit}}[i, j]$, since we are assuming no naturals are possible. For a single Ten, we use the formula:

$$E_{\text{hit}}[\text{single } 10, j] = \frac{1}{27}\Bigg(8E[10+10, j] + 2SE[10+1, j] + E[\text{JT}, j] + SE[10+2] + \sum_{k=2}^{9}(2E[10+k, j])\Bigg).$$

We know that any soft 1 must be an Ace, so we have

$$SE_{\text{hit}}[1, j] = \tfrac{1}{27}\left[8E[1+10, j] + 2SE[\text{AA}, j] + SE[1+2] + \sum_{k=2}^{9}(2SE[1+k, j])\right].$$

Since AA is a natural, we know all soft 2s must be Jokers, so we have

$$SE_{\text{hit}}[2, j] = \tfrac{1}{27}\left[8E[\text{JT}, j] + 2SE[2+1, j] + SE[i+2] + \textstyle\sum_{k=2}^{9}(2SE[2+k, j])\right].$$

Once we have the results from hitting and standing on a given total, we set

$$E[i, j] = \max(E_{\text{hit}}[i, j], E_{\text{stand}}[i, j])$$

and record whether the player should hit or stand. The results must be calculated by moving from high player totals to low player totals, for the same reason that we calculated banker distributions from high to low: Each answer depends only on the answers for higher totals. We can also calculate the favorability of the infinite deck game by taking the weighted average of the expectation over all possible banker upcards and player initial cards. The possible initial cards for the player include single 10, soft 1, soft 2, and hard 2 through 9.

Once we solved the infinite-deck game, we turned to Monte Carlo simulations of a finite deck. We dealt from a six-deck shoe and reshuffled whenever the deck had fewer than 120 cards.

Let p be the probability that the player wins (for the moment, we neglect ties). We are concerned with the player's advantage, $p - (1-p) = 2p - 1$, which we will write as a percentage and refer to as the player's *expectation*. For estimating the parameter p of a Bernoulli distribution, the Central Limit Theorem gives the half-width of a 95% confidence interval as $1.96\sqrt{p(1-p)/n}$ for a n random trials. Since the player's probability of winning p is very near one-half, the quantity $p(1-p)$ is approximately one-fourth, and the half-width

[2]For example, you cannot reach a natural if you have a 7 and a 3.

of the confidence interval is approximately $0.98/\sqrt{n}$. For $p-(1-p)=2p-1$, the confidence interval is twice as wide. Hence, for the one billion simulations that we did for each trial, the half-width of a 95% confidence interval for the player's expectation is $1.96/\sqrt{10^9}=0.00006$. In other words, our estimates of the player's expectation, in terms of percentages, should be accurate to two decimal places.

We removed cards and ran the simulation to see the effect on favorability. In each trial, we removed one deck's worth of a given denomination (2 Jokers, 4 Aces, 4 Twos, ..., 4 Nines, or 16 Tens) from the six-deck shoe. With a simulation of one billion hands, an observed difference between the stripped deck and the full deck's favorability is very likely not due to the randomness of the Monte Carlo simulations but to the difference in deck compositions. We took the difference between a full deck's expectation and the stripped deck's expectation, divided by the number of cards removed, and used this value as an approximation of a card's value. By doing this, we could see which cards we should bother counting and whether the deck fluctuates in favorability enough for bet variance to make the game profitable for the player.

Results

Table 1 indicates the player's optimal strategy for the player's hard total and the banker's upcard.

Table 1.
Optimal strategy given player's hard total and banker's upcard (infinite deck).
An "H" indicates that the player should hit and an "S" indicates that the player should stand.

	Joker	Ace	2	3	4	5	6	7	8	9	T
13	H	H	H	H	H	H	H	H	H	H	H
14	H	H	H	H	H	H	H	H	H	H	H
15	H	H	H	H	H	H	H	H	H	H	H
16	H	H	H	S	S	S	S	H	H	H	H
17	H	H	H	S	S	S	S	S	H	H	H
18	S	S	S	S	S	S	S	S	S	S	S
19	S	S	S	S	S	S	S	S	S	S	S
20	S	S	S	S	S	S	S	S	S	S	S
21	S	S	S	S	S	S	S	S	S	S	S
22	S	S	S	S	S	S	S	S	S	S	S

Table 2 gives the percentage expectation for a player who implements the basic strategy from **Table 1**, for various stripped decks. Each value was generated by a simulations of 10^9 hands.

- The first row gives the player's expectation (−1.29%) for a standard six-deck shoe.
- In the remaining rows, the second column gives the player's expectation for a six-deck shoe that is missing one-sixth of the cards of one denomination.

Thus, a player has an expectation of −1.90% for a six-deck shoe that is missing 16 Tens.

- The third column shows the deviation in percentage points of the expectation for the stripped deck from that for a standard six-deck shoe.
- The fourth column gives the the deviation divided by the number of cards removed.
- The fifth column shows the the ratio of each estimate to the smallest one. Removal of a Five from the deck has the greatest impact on a player's expectation, while removal of an Ace has the smallest effect.[3]

Table 2.

Effects of removing from the six-deck shoe one-sixth of the cards with a particular value; each result is based on simulation of 10^9 hands.

Card Removed	Percentage Expectation	Deviation from Expectation for a Full Six-Deck Shoe	Value per Card	Ratio
None	−1.294	0		
Joker	−1.265	0.028	0.0141	3.1
Ace	−1.275	0.018	0.0046	1
Two	−1.195	0.099	0.0247	5.4
Three	−1.169	0.124	0.0311	6.8
Four	−1.155	0.138	0.0346	7.6
Five	−1.127	0.167	0.0417	9.1
Six	−1.228	0.065	0.0164	3.6
Seven	−1.363	−0.069	−0.0173	−3.8
Eight	−1.219	0.074	0.0185	4.1
Nine	−1.423	−0.130	−0.0325	−7.1
Ten	−1.904	−0.611	−0.0382	−8.4

Tables 3–4 give the probability that the banker will reach a particular final total (top row) from a starting hard total (**Table 3**) or a starting soft total (**Table 4**). **Table 3** treats only starting totals between 18 and 27, since the banker cannot end with less than 18 (hitting is required) and more than 27 is impossible; **Table 4** treats only starting totals between 9 and 17, as these are the only soft totals on which the banker can legally stand.

Table 5 indicates a player's expectation for a hard card total (left column) and a given banker upcard (top row) for an infinite deck. The second row indicates the player's expectation for starting from hard 0 when the banker's upcard is visible but before the player has been dealt any cards. The row labeled "s10" indicates a single-card total of 10 (single 10), while "m10" indicates a multiple card total of 10 (multi 10). There is no row for a hard 1, as such a total is impossible. If we weight the average of a player's expectation at hard zero

[3]In standard Blackjack, Aces are valuable to the player, since a two-card total of 21 pays the player 3-to-2 odds. No such premium is paid in LA Blackjack.

Table 3.
Probability of banker final total, given a starting hard total (infinite deck).

	27	26	25	24	23	22	21	20	19	18
27	1	0	0	0	0	0	0	0	0	0
26	0	1	0	0	0	0	0	0	0	0
25	0	0	1	0	0	0	0	0	0	0
24	0	0	0	1	0	0	0	0	0	0
23	0	0	0	0	1	0	0	0	0	0
22	0	0	0	0	0	1	0	0	0	0
21	0	0	0	0	0	0	1	0	0	0
20	0	0	0	0	0	0	0	1	0	0
19	0	0	0	0	0	0	0	0	1	0
18	0	0	0	0	0	0	0	0	0	1
17	.2963	.0741	.0741	.0741	.0741	.0741	.0741	.0741	.1111	.0741
16	.0219	.3018	.0796	.0796	.0796	.0796	.0796	.0796	.0823	.1166
15	.0345	.0306	.3104	.0882	.0882	.0882	.0882	.0882	.0925	.0909
14	.0269	.0413	.0373	.0373	.0949	.0949	.0949	.0949	.0983	.0993
13	.0294	.0343	.0486	.0447	.3245	.1023	.1023	.1023	.1060	.1057
12	.0313	.0372	.0421	.0565	.0525	.3323	.1101	.1101	.1140	.1138
11	.0314	.0370	.0425	.0464	.0610	.1104	.3326	.1104	.1143	.1140
10	.0326	.0385	.0439	.0489	.0529	.0942	.1148	.3370	.1188	.1185
9	.0350	.0412	.0470	.0523	.0574	.0653	.1024	.1229	.3495	.1270
8	.0374	.0441	.0507	.0560	.0613	.0723	.0738	.1109	.1361	.3580
7	.1080	.0662	.0732	.0796	.0856	.1015	.1058	.1100	.1578	.1122
6	.0341	.1173	.0756	.0826	.0892	.1083	.1127	.1186	.1255	.1361
5	.0402	.0441	.1273	.0854	.0925	.1135	.1182	.1231	.1334	.1222
4	.0362	.0492	.0531	.1361	.0944	.1173	.1224	.1278	.1366	.1270
3	.0375	.0455	.0585	.0612	.1453	.1211	.1265	.1322	.1416	.1297
2	.0383	.0470	.0550	.0678	.0715	.1740	.1304	.1366	.1463	.1311

Table 4.
Probability of banker final total, given a starting soft total (infinite deck).

	27	26	25	24	23	22	21	20	19	18
17	.2963	.0741	.0741	.0741	.0741	.0741	.0741	.0741	.1111	.0741
16	.0219	.3018	.0796	.0796	.0796	.0796	.0796	.0796	.0823	.1166
15	.0345	.0306	.3104	.0882	.0882	.0882	.0882	.0882	.0925	.0909
14	.0269	.0413	.0373	.3172	.0949	.0949	.0949	.0949	.0983	.0993
13	.0294	.0343	.0486	.0447	.3245	.1023	.1023	.1023	.1060	.1057
12	0	0	0	0	0	1	0	0	0	0
11	0	0	0	0	0	0	1	0	0	0
10	0	0	0	0	0	0	0	1	0	0
9	0	0	0	0	0	0	0	0	1	0

by the frequency of the dealer's upcard, we get the expectation for the infinite deck game, −1.33%.

Table 5.

Player expectation given a hard hand value and a banker upcard (infinite deck).

	Joker	A	2	3	4	5	6	7	8	9	T
0	−.3661	−.2955	−.0461	.0460	.0768	.1096	.1331	.1591	.1041	.0414	−.0814
2	−.3476	−.2435	−.0171	.0721	.1022	.1343	.1567	.1813	.1215	.0633	−.0475
3	−.4129	−.3334	−.0972	−.0081	.0276	.0619	.0853	.1113	.0284	−.0305	−.1368
4	−.4299	−.3529	−.1221	−.0389	−.0031	.0353	.0588	.0855	−.0007	−.0578	−.1608
5	−.4481	−.3736	−.1494	−.0679	−.0362	.0022	.0299	.0573	−.0314	−.0870	−.1869
6	−.4683	−.3972	−.1783	−.0972	−.0622	−.0248	−.0020	.0259	−.0670	−.1203	−.2161
7	−.4849	−.4183	−.2009	−.1080	−.0727	−.0352	−.0122	.0144	−.0808	−.1415	−.2417
8	−.5175	−.4712	−.2161	−.1124	−.0771	−.0397	−.0166	.0102	−.1280	−.1884	−.2847
9	−.4354	−.3851	−.0969	.0055	.0375	.0711	.0984	.1218	.0976	−.0161	−.1697
s10	−.32	−.2649	.0163	.1135	.1419	.1721	.1957	.2233	.2001	.1395	−.0141
m10	−.3451	−.2943	−.0185	.0789	.1073	.1375	.1611	.1888	.1656	.1051	−.0473
11	−.2588	−.1370	.0823	.1786	.2053	.2331	.2543	.2785	.2488	.1993	.0967
12	−.2137	−.0425	.1475	.2305	.2553	.2820	.3008	.3220	.2788	.2342	.1328
13	−.4793	−.4130	−.1771	−.0930	−.0461	−.0104	.0127	.0397	−.0901	−.1408	−.2290
14	−.5195	−.4580	−.2407	−.1767	−.1327	−.0821	−.0582	−.0290	−.1575	−.2044	−.2860
15	−.5588	−.5011	−.3045	−.2429	−.2162	−.1690	−.1298	−.0987	−.2241	−.2674	−.3430
16	−.5989	−.5469	−.3660	−.3021	−.2622	−.2209	−.2023	−.1702	−.2949	−.3341	−.4025
17	−.6246	−.5762	−.4106	−.3021	−.2622	−.2209	−.2023	−.1745	−.3284	−.3726	−.4439
18	−.7199	−.7218	−.4408	−.3021	−.2622	−.2209	−.2023	−.1745	−.5021	−.5342	−.5666
19	−.4528	−.4545	−.0283	.0988	.1284	.1570	.1953	.2076	.3500	.0693	−.2106
20	−.1585	−.1597	.2546	.3727	.3928	.4136	.4394	.4755	.5969	.5417	.2452
21	.1296	.3512	.5216	.6314	.6429	.6549	.6706	.6913	.7816	.7670	.6969
22	.3395	.7837	.8260	.8789	.8827	.8865	.8917	.8985	.9277	.9347	.8688
JT	.7037	.8519	1	1	1	1	1	1	1	1	.9630
AA	1	.9259	1	1	1	1	1	1	1	1	1
23	−.7568	−.7606	−.5124	−.4474	−.3565	−.3134	−.2915	−.2602	−.5634	−.5916	−.6194
24	−.8282	−.8309	−.6516	−.6548	−.5870	−.4914	−.4634	−.4254	−.6807	−.7013	−.7212
25	−.8901	−.8903	−.7743	−.7754	−.7761	−.7041	−.6216	−.5783	−.7869	−.8007	−.8140
26	−.9408	−.9410	−.8763	−.8794	−.8784	−.8755	−.8145	−.7177	−.8812	−.8889	−.8963
27	−.9819	−.9819	−.9617	−.9625	−.9638	−.9598	−.9659	−.8920	−.9626	−.9650	−.9674

Does It Pay to Count Cards?

Counting Tens

In standard Blackjack, a high proportion of Tens in the deck is favorable to the player, and counting Tens and adjusting bets accordingly gives the player a positive expectation. **Table 2** shows that in LA Blackjack, too, Tens are favorable to the player.

We fed the simulator decks with increasing numbers of Tens in them until we found the density (about 40%) at which the player's expectation is 0. We had the player count Tens and increase the bet sixfold (the maximum multiple allowed by the rules if the player bets in intervals of $100) whenever the deck is favorable. The results from the simulation: The player still loses 1.29% (cf. the expectation for the infinite deck, −1.33%). In fact:

In 10^9 hands, a Ten-counting player never had an opportunity to raise the bet.

Counting "Good" and "Bads"

What if we count other cards too? Removing a single Ten, Nine, or Seven (there are 96 + 24 + 24 = 144 such cards) decreases the player's expectation by about 0.03%; removing a single Two, Three, Four, Five, Six, or Eight (there are 6 × 24 = 144 such cards) helps the player by about the same amount; and removing a Joker or an Ace has little effect on deck favorability.

To compensate for the 1.33% negative expectation plus another 1% to cover the house's fees, the deck would need favorability to increase by 2.3%; that would require 77 more "good" cards removed than "bad" cards. Since we deal only 204 cards before reshuffling, we need to draw good cards at least twice as fast as we draw bad cards for the deck ever to get favorable.

We estimate the probability ϵ that a deal will offer a favorable situation for the player, using simulations and the assumption that the deck is composed half of cards good for the player and half of cards bad for the player. We keep track of the number of good cards and bad cards that remain in the deck. If the deck ever reaches a point where the number of good cards exceeds the number of bad cards by more than 77, the deck is favorable to the player. In a simulation of 10^9 hands, the deck never reached a favorable position for the player. Based on these results, we estimate that $\epsilon < 10^{-6}$, that is:

A favorable deck comes around less often than once in a million plays.

Since the minimum bet is $10 and the maximum bet is $600, the player will find it impossible to recover the losses incurred during the majority of play.

Final Analysis

In standard Blackjack, the house has a 5.5% advantage if the player imitates the dealer's strategy. The player can compensate for this disadvantage by making intelligent decisions and counting cards.

In LA Blackjack, the player starts with less of a disadvantage, losing 1.7% (apart from betting fees) when mimicking the dealer; but intelligent decisions and bet variance help the player little:

- Following a basic strategy gains 0.4% for the player, for a net of −1.3%.
- Counting cards gets the player almost nothing, as opportunities to deviate productively from the basic strategy are virtually nonexistent.
- Bet variance is completely ineffective, because favorable decks are virtually nonexistent.

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References

Benjamin, Arthur T., and Eric Huggins. 1993. Optimal Blackjack strategy with "Lucky Bucks." *The UMAP Journal* 14(4): 309–318.

Griffin, Peter A. 1996. *The Theory of Blackjack*. Las Vegas, NV: Huntington Press.

About the Authors

Arthur Benjamin is an Associate Professor of Mathematics at Harvey Mudd College, where he has taught since earning his Ph.D. in Mathematical Sciences from Johns Hopkins University in 1989. He is editor of the Spectrum book series for the Mathematical Association of America (MAA), and Associate Editor of *Mathematics Magazine* and of *The UMAP Journal*. Presently he is the all-time point leader of the American Backgammon Tour. In addition (and multiplication), he performs as a lightning calculator and "mathemagician." Recently, he received the Distinguished Teaching Award from the Southern California Section of the MAA.

Michael Lauzon attends Harvey Mudd College and will receive a B.S. in Mathematics in May 2000. His mathematical interests are topology, analysis, and, of course, analyzing any game that he is presented with. He also enjoys, bridge, chess, backgammon, and baseball.

Chris Moore attends Harvey Mudd College and will receive an undergraduate degree in mathematics and computer science in May 2000. His academic interests include combinatorics, algorithms, and virtually any subject related to theoretical computer science. During his free time, Chris enjoys playing chess, bridge, backgammon, or whatever game he can convince somebody to play.