



Analyzing Immunotherapy and Chemotherapy of Tumors
through Mathematical Modeling
Summer Student-Faculty Research Project

by
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Abstract

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The development of immunotherapy in treating certain forms of cancer has recently become an exciting new focus in cancer research. In some preliminary studies, immunotherapy has been found to be most effective when administered in conjunction with chemotherapy [89]. Precisely how various types of immunotherapy work, and how they should optimally be administered, either alone or in conjunction with chemotherapy, is not yet well understood. We propose to contribute to the emerging body of cancer treatment research by developing and analyzing new mathematical models of cancer treatment that include vaccine therapy, activated anticancer-cell transfer, and activation-protein injections in combination with chemotherapy. We build on existing models that are already successfully developed. Results of our model simulations are validated by comparing results from mouse [46] and human [49] data. The mathematical models we develop will enrich the study of cancer treatment and aid in hastening progress toward an increased understanding and more widespread availability of this new type of this new combination approach to cancer therapy.

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