Math 25B: Calculus
Homework 3

Due Friday, October 29, 2010

1. Let $f(x)$ be a function of a single variable and let $c \in \mathbb{R}$ be a given constant. Prove, starting from the definition, that if $g(x) = cf(x)$, then $g'(x) = cf'(x)$.

2. Use the rules of differentiation (i.e., sum, product, quotient, power, and chain rule) to find $f'(x)$ for each of the following functions:

   (a) $f(x) = \sin x + \sin x^2$.

   (b) $f(x) = \dfrac{\sin(\cos x)}{x}$.

   (c) $f(x) = \sin(\cos(\sin x))$.

   (d) $f(x) = \sin\left(\dfrac{x^3}{\cos x^3}\right)$.

3. Find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ for each of the following:

   (a) $f(x, y) = \cos(xy) - \sin 2y$

   (b) $f(x, y) = e^{-(x-2y)^2}$

4. Consider the function $u(x, t) = e^{-\lambda k^2 t} \sin(\sqrt{\lambda} x)$. By computing the partial derivatives $\frac{\partial u}{\partial t}$ and $\frac{\partial^2 u}{\partial x^2}$, verify that the function $u(x, t)$ satisfies the partial differential equation

$$\frac{\partial u}{\partial t} = k^2 \frac{\partial^2 u}{\partial x^2}.$$