DAGAN KARP



# ALGEBRAIC GEOMETRY FALL 2016

## I. SYLLABUS

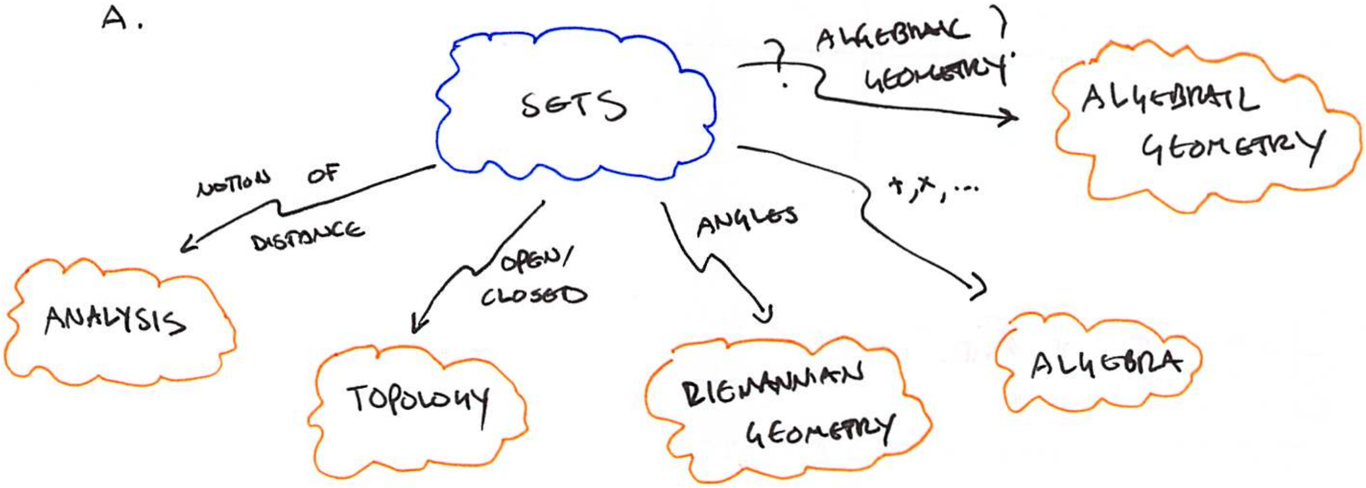
A. DISTRIBUTE + DISCUSS

(1) GIVE URL FOR COURSE HOMEPAGE : MATH.HMC.EDU/~dk/MATH176

(2) SPEND TIME DISCUSSING DISABILITY RESOURCE CENTER, PROTESTS, CRITICAL READINGS, HOMEWORK.

## II. INTRODUCTION TO ALGEBRAIC GEOMETRY

A.




B. HISTORY: WHAT IS ALGEBRAIC? AND WHAT IS GEOMETRIC?

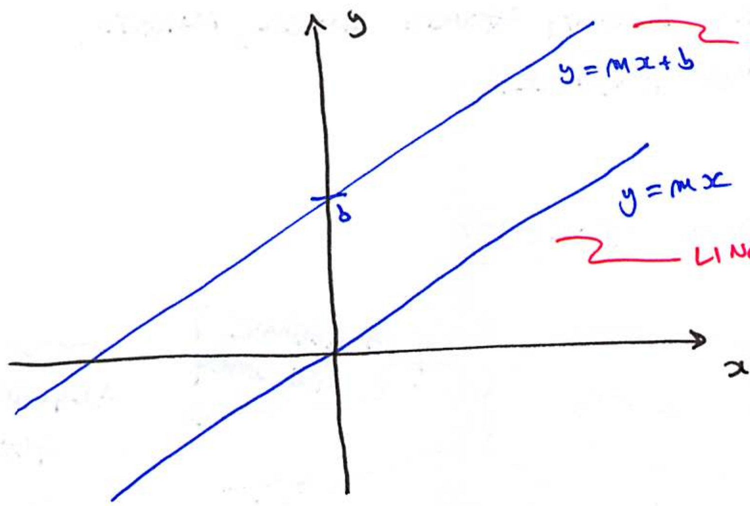
(1) MUHAMMAD AL KHWARIZMI (ALGORITHM): PERSIAN MATHEMATICIAN + ASTRONOMER WORKING IN BAGHDAD DURING ABBASID CALIPHATE, CIRCA 780-850. DISCOVERED TECHNIQUES TO SOLVE QUADRATIC EQ'S. ONE WAS AL JABR. → ALGEBRA (COLONIAL HISTORY OF THE MID. EAST)

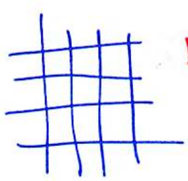
$$\boxed{\text{ALGEBRAIC = RELATED TO POLYNOMIALS}}$$

(2) WHAT IS GEOMETRIC? MANY ANSWERS TO THIS QUESTION, DEPENDING ON PERSPECTIVE. WE BEGIN THIS COURSE BY CONSIDERING GEOMETRIC OBJECTS AS THOSE WHO LIVE IN AFFINE OR

NEXT CLASS → PROJECTIVE SPACE.

(a) AFFINE SPACE!



$\mathbb{R}^2$ v $\mathbb{A}^2_{\mathbb{R}}$ ~ GEOMETRIC

GROUP, VECTOR SPACE, ETC...
(ALGEBRAIC...)

(3) HOW TO COMBINE ALGEBRA + GEOMETRY: ALGEBRAIC SETS!

DEF: A SUBSET $X \subseteq \mathbb{A}^n_{\mathbb{R}}$ $(= \mathbb{R}^n)$ IS AN ALGEBRAIC SET IF $X$ IS THE ZERO LOCUS OF A COLLECTION OF POLYNOMIALS $(f: \mathbb{R}^n \to \mathbb{R})$, IE $\exists$ A COLLECTION $\{f_\alpha\}_{\alpha \in I}$ SUCH THAT

$$X = \{ p \in \mathbb{A}^n_{\mathbb{R}} \mid f_\alpha(p) = 0 \text{ FOR ALL } \alpha \in I \}.$$

NOTATION: IF $f$ IS A POLYNOMIAL IN $(x_1, x_2, .., x_n)$, IE

$f \in \mathbb{R}[x_1, .., x_n]$, THE ZEROS OF $f$ ARE DENOTED

$$Z(f) = \{(a_1, .., a_n) \in \mathbb{A}^n_{\mathbb{R}} \mid f(a_1, .., a_n) = 0\}.$$

SIMILARLY, FOR ANY COLLECTION OF POLYNOMIALS $\{f_\alpha\}_{\alpha \in I}$,

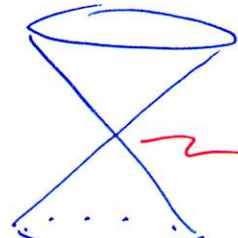
$$Z(\{f_\alpha\}) = \{(a_1, .., a_n) \in \mathbb{A}^n_{\mathbb{R}} \mid f_\alpha(a_1, .., a_n) = 0 \ \forall \alpha \in I\}.$$

NOTE: REID USED $V(f)$ FOR VARIETY, I USE $Z(f)$ FOR ZEROS, BUT $\underbrace{V(f) = Z(f)}_{\text{EITHER IS FINE}}$.

SOON, MAYBE NEXT CLASS.

EX 1: $f(x,y) = x^2 + y^2 - 1$

EX 2: $g(x,y,z) = x^2 + y^2 - z^2$




EX 3: $h(x,y,z) = x^3 + y^3 - z^3$

FERMAT!