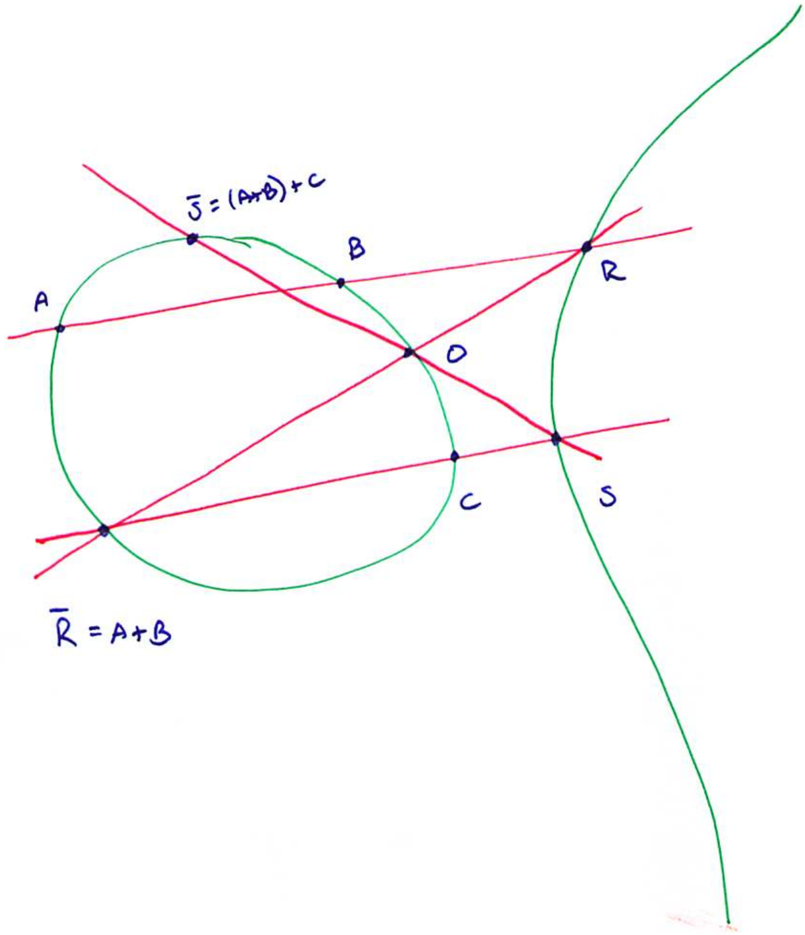
DAVAN KARP

A. G. CLASS 18



(1) WELL DEFINED.

IF $P \neq Q$, $L = PQ \subseteq \mathbb{P}^2_k$ IS WELL DEFINED. THEN $F|_L$ IS A FORM IN 2 VARIABLES WITH 2 GIVEN ZEROS IN k. IT THEREFORE SPLITS AS A PRODUCT OF 3 LINEAR FACTORS AND THE THIRD POINT OF INTERSECTION R HAS COORD'S IN k.

IF $P = Q$, THEN BY ASSUMPTION $\exists$ UNIQUE $L \subseteq \mathbb{P}^2_k$ s.t. $F|_L$ HAS A REPEATED ROOT AT P. SO R IS DETERMINED AGAIN AS ABOVE.

$P=Q$, $P=R$, $Q=R$, $P=Q=R$ ALL ALLOWED, ONLY

CHANGED MULTIPLICITY OF ROOT OF FIL (TANGENT AND

INFLECTION POINTS)

(2) $A+O = A$

(3) $A+B = B+A$

(4) $-A \in C$: $-A = \overline{OA} \cap C$

(5) $(A+B)+C = A+(B+C)$.

LET $(A+B)+C = \overline{S}$ $\quad (B+C)+A = \overline{T}$

DEFINE

$L_1 = ABR \quad L_2 = RO\overline{R} \quad L_3 = C\overline{R}S \quad L_4 = SO\overline{S}$

$M_1 = BCQ \quad M_2 = QO\overline{Q} \quad M_3 = A\overline{Q}T \quad M_4 = TO\overline{T}$

CONSIDER THE CUBICS

$D_1 = L_1 \cup M_2 \cup L_3$

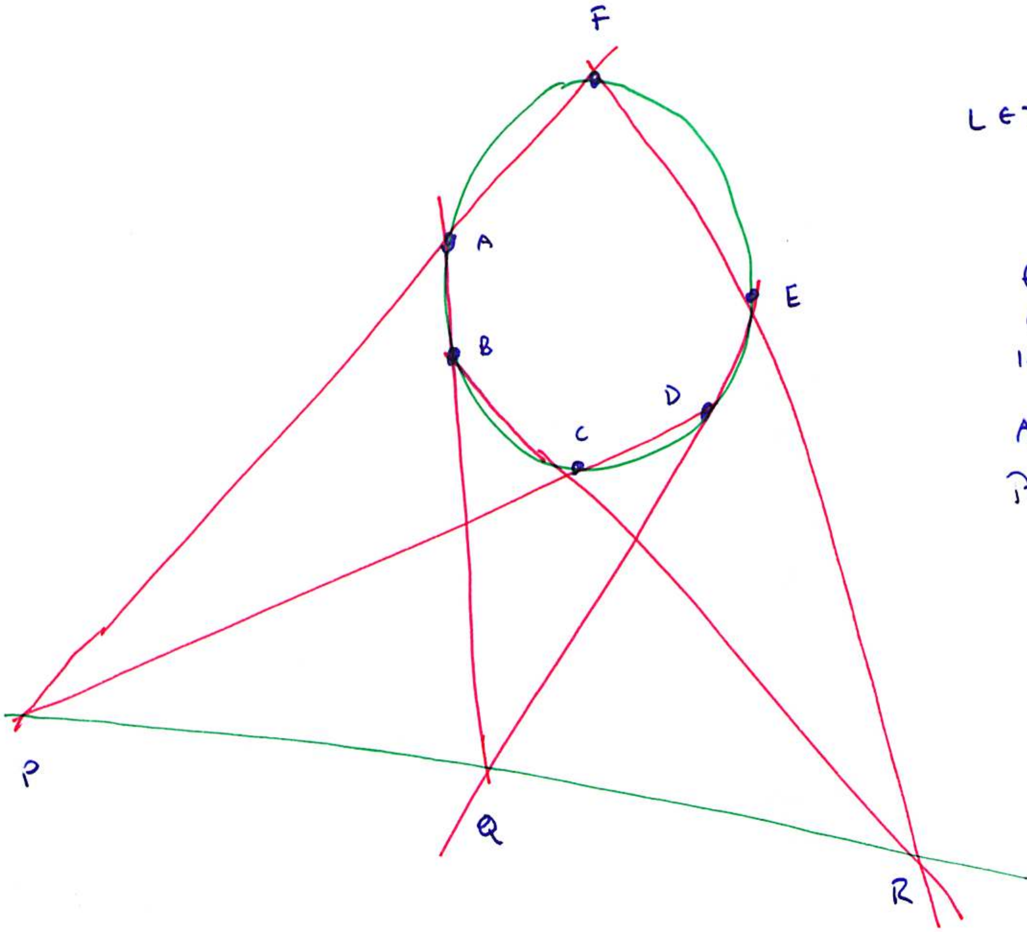
$D_2 = M_1 \cup L_2 \cup M_3$

$C \cap D_1 = \{A, B, C, O, R, \overline{R}, Q, \overline{Q}, S\}$

$C \cap D_2 = \{A, B, C, O, R, \overline{R}, Q, \overline{Q}, T\}$

IF $A, B, C, O, R, \overline{R}, Q, \overline{Q}, S$ DISTINCT, $\Rightarrow S = T$.

THM THE MYSTIC HEXAGON / PASCAL'S THM



LET $ABCDEF \subseteq \mathbb{P}^2_{\mathbb{R}}$

FORM A HEXAGON.

EXTEND LINES AB, BC, ..., EF, FA
OPPOSITE SIDES
INTERSECTING IN P, Q, R.

ASSUME ALL PTS DISTINCT.
THEN ABCDEF CONCONIC

$\Longleftrightarrow$

PQR COLLINEAR.

PF: $L_1 = PAF$ $\quad L_2 = QDE$ $\quad L_3 = RBC$

$M_1 = PCD$ $\quad M_2 = QAB$ $\quad M_3 = REF$

$C_1 = L_1 + L_2 + L_3$ $\qquad C_2 = M_1 + M_2 + M_3$ CUBICS

$$C_1 \cap C_2 = \{A, B, C, D, E, F, P, Q, R\}$$

PQR COLLINEAR $L = PQR$. LET $\tilde{C}$ BE CONIC THRU
ABCDE. THEN $L + \tilde{C}$ IS A CUBIC THRU 8 PTS
ABCDE PQR

THUS $F \in \tilde{C} + L$. BUT $F \notin L$ BY ASSUMPTION. SO $F \in \tilde{C}$.

CONVERSELY, ABCDEF ON CONIC $\tilde{C}$, LET $L = PQ$.

THEN $L \cup \tilde{C}$ IS A CUBIC THRU A, B, C, D, E, F, P, Q

$\Rightarrow$ $R \in L \cup \tilde{C}$. $R \notin \tilde{C}$ ($\tilde{C}$ NOT LINE PAIR)

$\Rightarrow$ $R \in L$. QED