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A.G. CLASS 21

DEF: BLOWUP OF $\vec{0} \in \mathbb{C}^n = \{(z_1, \dots, z_n)\}$.

$$Bl_{\vec{0}}\, \mathbb{C}^n = \{((z_1, \dots, z_n), (y_1 : \dots : y_n)) \in \mathbb{C}^n \times \mathbb{CP}^{n-1} \mid z_i y_j = z_j y_i \quad \forall i,j\}$$

$\pi^{-1}(0)$ IS CALLED THE EXCEPTIONAL DIVISOR OF THE BLOWUP.

THE PROJECTION MAP $\pi : Bl_{\vec{0}}\, \mathbb{C}^n \to \mathbb{C}^n$, $\pi(z,y) = z$.

GROUPWORK: (1) FIND $\pi^{-1}(0)$, FOR $z \neq 0$, $\pi^{-1}(z)$.

(2) USE THIS TO FIND $Bl_{(1:0:0)}\, \mathbb{CP}^2$.

DEF: LET $V \subseteq \mathbb{C}^n$ BE THE LOCUS $V = Z(z_{k+1}, z_{k+2}, \dots, z_n)$

((COORDINATE PLANE)).

$$Bl_V\, \mathbb{C}^n = \{((z_1, \dots, z_n), (l_{k+1} : \dots : l_n)) \in \mathbb{C}^n \times \mathbb{P}^{n-k-1} \mid z_i l_j = z_j l_i, \ k+1 \leq i,j \leq n\}$$

THE PROPER TRANSFORM OF $Y \subseteq \mathbb{C}^n$ IS

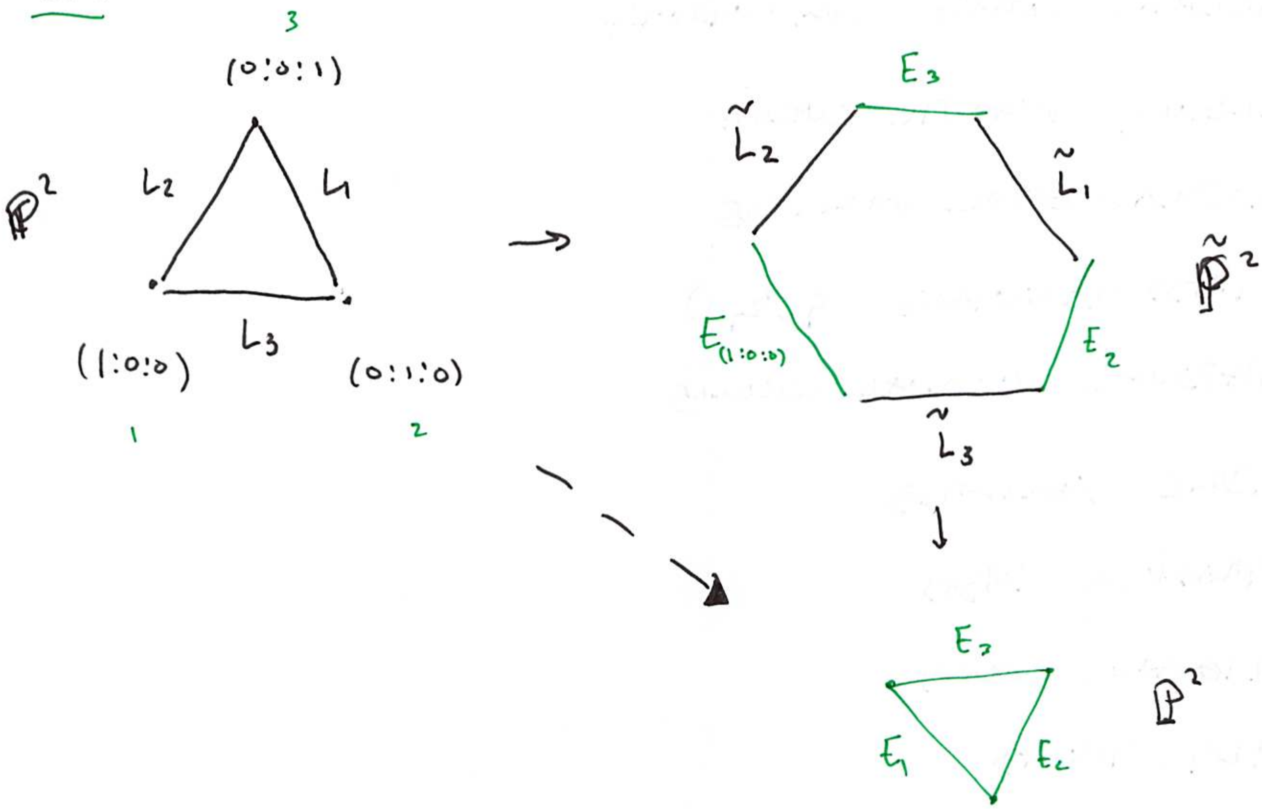
$$\overline{\pi^{-1}(Y \setminus V)} = \overline{\pi^{-1}(Y) \setminus E}$$

NOTE: FOR ANY VECTOR SPACE V, $\mathbb{P}(V)$ = ONE DIM'L VECTOR SUBSPACES. $\mathbb{P}(\mathbb{C}^{n+1}) = \mathbb{CP}^n$

GROUPWORK: $\mathbb{P}(T_p \mathbb{C}^n) \cong E$

DEF: A BIRATIONAL MAP $\mathbb{CP}^2 \to \mathbb{CP}^2$ IS CALLED A CREMONA TRANSFORMATION.

EX:



(QUAD. TRANSFORMATIONS)

$$(x_0 : x_1 : x_2) \mapsto (x_1 x_2 : x_0 x_2 : x_0 x_1)$$

$$= \left(\frac{1}{x_0} : \frac{1}{x_1} : \frac{1}{x_2}\right) \text{ on } \{U\} = \{x_i \neq 0\}.$$