TROPICAL GEOMETRY SPRING 2013
HOMEWORK 5

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(1) Let ν_p be the p-adic valuation. Prove, as is claimed in Example 2.1.2, that the residue field of ν_p on $\mathbb{Q}$ is isomorphic to $\mathbb{Z}/p\mathbb{Z}$.

(2) Let $\nu : K \to G \cup \{\infty\}$ be a valuation on the field K. The set of all field elements with non-negative valuation is called the *valuation ring* of ν and is denoted R_K;

$$R_K = \{c \in K : \nu(c) \geqslant 0\}.$$

Prove the following.

 (a) R_K is a subring of K and K is the field of fractions of R_K. In fact, for every $c \in K$, we have $c \in R_K$ or $c^{-1} \in R_K$.

 (b) The group of units of R_K is $U_K = \{c \in K : \nu(c) = 1\}$.

 (c) The ring R_K has exactly one maximal ideal (ie R_K is a local ring), given by

$$\mathfrak{m}_K = \{c \in K : \nu(c) > 0\},$$

 which contains all the elements of R_K which are not units in R_K.

 (d) The ideals of R_K form a chain, ie if I and J are ideals of R_K, then $I \subseteq J$ or $J \subseteq I$.

(3) Solve problem 2.7.4.

Date: due Thur Feb 28.